An Analysis of Three Self-Balancing Phase Inverters*

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Summary—**A self-balancing phase inverter is a circuit converting one driving voltage to two output voltages of opposite phase but of essentially equal magnitude by an inherent characteristic of the device and not by virtue of any critical adjustment. The algebraic solution of three self-balancing phase inverters is given, assuming all circuit elements are linear. Included in the solution are the conditions for self-balance, the balance ratio, and the voltage gain. From this information, the type of inverter for a particular service may be selected and designed.**

Introduction

THE DESIGN of audio and video amplifiers frequently requires the conversion from a single-ended to a double-ended (or push-pull) channel. Two familiar examples of this type of amplifier are the driving of cathode-ray-tube plates in push-pull from an amplifier that must be, for convenience, single ended at its input; and the driving of audio power amplifiers in push-pull where it is again convenient to use single-ended voltage-amplifier stages and input.

This conversion requires a network with two output voltages, equal in magnitude but opposite in phase, and proportional to its driving voltage. While a transformer fulfills these requirements, it is frequently more economical and more uniform in frequency response to use a tube to invert the phase in a circuit that automatically equalizes the two output signals.[1,2]

Three self-balancing phase-inverting circuits will be analyzed: the common-plate-impedance inverter, the common-cathode-impedance inverter, and the cathode- and plate-loaded inverter. The only assumption made in the analysis is that all circuit elements (and in particular, the tube parameters) are linear. This assumption is well justified, as this type of amplifier is generally designed for linear operation. And, although the solutions are given for such a frequency range that the load impedances are resistive, the solution could be extended to any frequency by the substitution of their complex impedances. The circuits in Figs. 1, 2, 3, 4, 5, and 6 are all equivalent signal-voltage circuits, as there is no loss in generality by omitting the direct-current components.

Common-Plate-Impedance Self-Balancing Inverter

Referring to Figs. 1 and 2, the following tube parameters are defined as:

$$r_{p1} = \frac{\partial E_{p1}}{\partial i_1} \qquad \mu_1 = \frac{\partial E_{p1}}{\partial e_g}$$

$$r_{p_2} = \frac{\partial E_{p_2}}{\partial i_2} \qquad \mu_2 = \frac{\partial E_{p_2}}{\partial e_0}.$$

Then, adding the voltages around three closed circuits,

$$\mu_1 e_g - i_1 r_{p1} - i_1 r_1 - (i_1 - i_2)R_0 = 0$$

$$\mu_2 e_0 - i_2 r_{p_2} - i_2 r_2 + (i_1 - i_2)R_0 = 0$$

$$e_0 - (i_1 - i_2)R_0 = 0$$

and as

$$E_2 = i_2 r_2 - e_0$$

and

$$E_1 = i_1 r_1 + e_0.$$

Solving for E_2/E_1

$$\frac{E_2}{E_1} = \frac{\mu_2 R_0 r_2 - r_{p_2} R_0}{\mu_2 R_0 r_1 + r_1 r_2 + r_1 r_{p_2} + R_0 r_1 + R_0 r_2 + R_0 r_{p_2}}. \qquad (1)$$

In (1) the terms $\mu_2 R_0 r_2$ and $\mu_2 R_0 r_1$ are much greater than the rest, and it can be seen that, if the others were

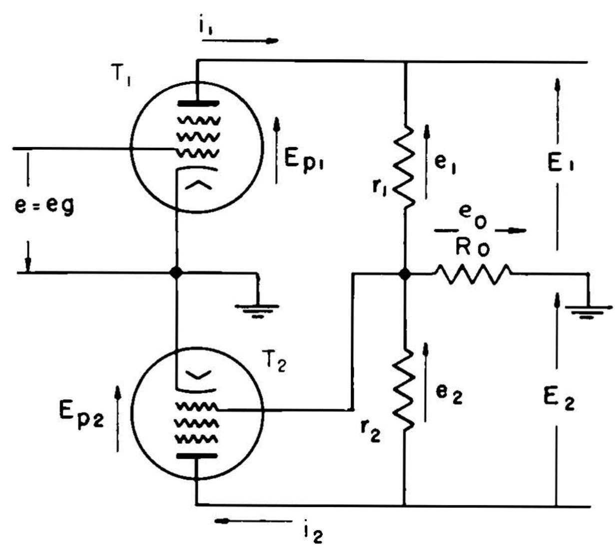

Fig. 1—Equivalent signal-voltage circuit of common-plate-impedance self-balancing inverter.

negligible compared with these two, the condition for balance (equal output voltages) would be

$$\mu_2 R_0 r_2 = \mu_2 R_0 r_1 \quad \text{or} \quad r_2 = r_1.$$

The circuit could be brought to balance, however, in

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[1] J. G. Brainerd, "Ultra-High-Frequency Techniques," D. Van Nostrand Co., Inc., New York, N. Y., 1942, pp. 100–101.
[2] M.I.T. Staff, "Applied Electronics," John Wiley and Sons, New York, N. Y., 1943, pp. 489–490.

any event by adjusting r_1 and r_2 so that

$$\frac{E_2}{E_1} = 1$$

but we are more interested in the degree of unbalance resulting when $r_2 = r_1$, which we shall call r; then

$$\frac{E_2}{E_1} = \frac{\mu_2 R_0 r - r_{p_2} R_0}{\mu_2 R_0 r + r^2 + r r_{p_2} + 2R_0 r + R_0 r_{p_2}}. \tag{2}$$

In order to minimize the unbalance, R_0 should be selected as great as possible, as it can be seen from (2) that

$$\frac{E_2}{E_1} \rightarrow \frac{\mu_2 r - r_{p_2}}{\mu_2 r + 2r + r_{p_2}} \tag{3}$$

$$\lim R_0 \rightarrow \infty$$

which is the closest E_2/E_1 comes to 1 as R_0 varies from 0 to ∞. Practical limits, however, set the maximum value of R_0 near r. As a basis of comparison of this type of phase inverter with some others, let $R_0 = r$. Then

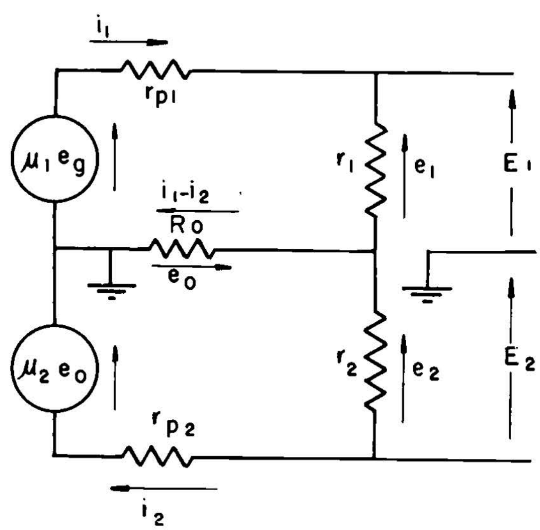

Fig. 2—Equivalent linear-tube-parameter circuit of common-plate-impedance self-balancing inverter.

$$\frac{E_2}{E_1} = \frac{\mu_2 r - r_{p_2}}{\mu_2 r + 3r + 2r_{p_2}}. \tag{4}$$

To simplify this expression further, if we define the gain of the inverter tube T_2 as N, and omit intermediate algebraic steps

$$N = \frac{E_2}{e_0} = \frac{\mu_2 r_2 - r_{p_2}}{r_{p_2} + r_2} \tag{5}$$

then with $r_1 = r_2 = r$ as above from (2)

$$\frac{E_2}{E_1} = \frac{N R_0}{r + R_0(N+2)} \tag{6}$$

and with $R_0 = r$ as in (4)

$$\frac{E_2}{E_1} = \frac{N}{N+3} \text{ (exactly)} \tag{7}$$

where N, the gain of the inverter tube T_2, is defined exactly by (5). It is essentially, however, the gain of T_2 as a normal plate-loaded voltage amplifier.

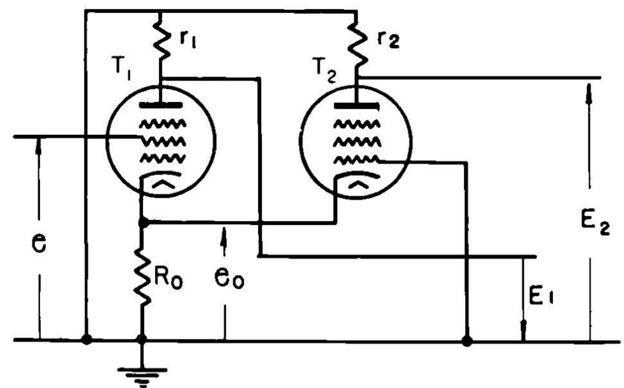

Fig. 3—Equivalent signal-voltage circuit of common-cathode-impedance self-balancing inverter.

That is

$$N \cong \text{normal plate-loaded gain} = \frac{\mu_2 r_2}{r_{p_2} + r_2}$$

as can be seen from (5), because generally

$$\mu_2 r_2 \gg r_{p_2}.$$

Let us stop to interpret these equations. We find from (7) that the balance of the inverter depends upon the gain of T_2, provided R_0 is sufficiently great as shown in (3), where the gain of T_2 is defined by (5). The balance also requires that the plate-load resistors of the two stages be equal as seen in (1), but the two tubes need not be the same. The tube T_1, then, acting only as a voltage amplifier has a gain

$$\frac{E_1}{e} = \frac{\mu_1\mu_2 R_0 r_1 + \mu_1(r_1 r_2 + r_1 r_{p_2} + r_1 R_0 + R_0 r_2 + R_0 r_{p_2})}{(r_1 + r_{p_1})(r_2 + r_{p_2}) + R_0(r_1 + r_{p_1})(1+\mu_2) + R_0(r_2 + r_{p_2})}. \tag{8}$$

The only purpose of writing (8) was to show approximately that the gain of T_1 is the same as a normal plate-loaded amplifier with a load resistance of r_1, if

$$\gg 1$$

for then

$$\frac{E_1}{e_0} \cong \frac{\mu_1\mu_2 R_0 r_1}{(\mu_2 + 1)(r_1 + r_{p_1})R_0} \cong \frac{\mu_1 r_1}{(r_1 + r_{p_1})}. \tag{9}$$

Common-Cathode-Impedance Self-Balancing Inverter

Using the same symbols as in the previous analysis, it can be seen from Fig. 3 that

$$e_{g1} = e - (i_1 + i_2)R_0$$
$$e_{g2} = (i_1 - i_2)R_0.$$

Adding the voltages around two closed paths in Fig. 4

$$(i_1 - i_2)R_0 - \mu_1[e - (i_1 - i_2)R_0] + i_1(r_1 + r_{p1}) = 0$$
$$(i_1 - i_2)R_0 + \mu_2(i_1 - i_2)R_0 - i_2(r_{p2} + r_2) = 0.$$

Then solving for E_2/E_1

$$\frac{E_2}{E_1} = \frac{R_0(\mu_2 + 1)r_2}{R_0(\mu_2 + 1)r_1 + (r_{p2} + r_2)r_1}. \qquad (10)$$

As in the previous solution, let

$$r_1 = r_2 = r.$$

R_0 should again be as great as practical limitations permit to obtain the best balance. In general this is near the value of r, and to compare this inverter with the previous type, let

$$R_0 = r_1 = r_2 = r.$$

Then

$$\frac{E_2}{E_1} = \frac{\mu_2 r + r}{\mu_2 r + 2r + r_{p2}}. \qquad (11)$$

This expression may also be simplified in the same manner as the previous case.

The common-cathode-impedance phase inverter, it has been seen, is quite analogous to the common-plate-impedance phase inverter. Good balance depends upon equal load resistors, high gain in T_2, and a common cathode resistor as large as practicable. The degree of balance is a little better than the common-plate-impedance type, but the voltage amplification of T_1 is found to be about half, for

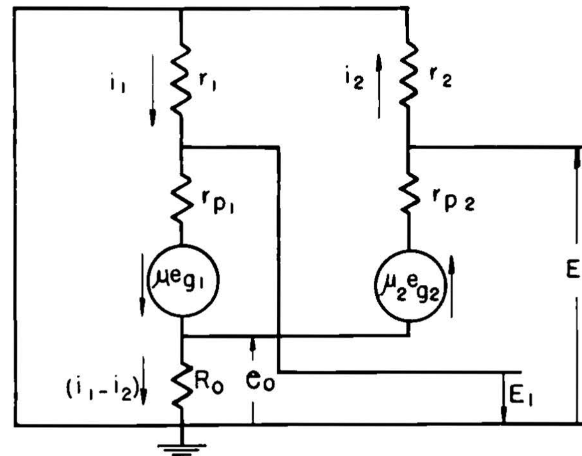

Fig. 4—Equivalent linear-tube-parameter circuit of common-cathode-impedance self-balancing inverter.

$$\frac{E_1}{e} = \frac{\mu_1(1 + \mu_2)R_0 r_1 + \mu_1(r_{p2} + r_2)r_1}{(\mu_2 + 1)(r_1 + r_{p1})R_0 + (\mu_1 + 1)(r_2 + r_{p2})R_0 + (r_1 + r_{p1})(r_2 + r_{p2})}. \qquad (15)$$

Define the gain of T_2 as

$$N = \frac{E_2}{e_0} = \frac{(\mu_2 + 1)r_2}{r_{p2} + r_2}. \qquad (12)$$

Then

$$\frac{E_2}{E_1} = \frac{R_0 N}{R_0 N + r} \qquad (13)$$

and with

$$R_0 = r$$
$$\frac{E_2}{E_1} = \frac{N}{N + 1} \text{ (exactly)} \qquad (14)$$

where N is again essentially the gain of T_2 as a normal plate-loaded voltage amplifier.

That is,

$$N \cong \text{normal plate-loaded gain} = \frac{\mu_2 r_2}{r_{p2} + r_2}$$

as it can be seen from (10), because generally

$$\mu_2 \gg 1.$$

To simplify the equation consider

$$r_1 + r_{p2} = r_2 + r_{p2}$$

and

$$\mu_1 = \mu_2$$

where

$$\mu_2 \gg 1.$$

Then

$$\frac{E_1}{e} = \frac{\mu_1 r_1}{2(r_1 + r_{p1})}. \qquad (16)$$

Again the only purpose of writing (15) was to show approximately the gain of T_1, which is roughly one half of the gain of the normal plate-loaded amplifier with a load resistance of r_1.

Cathode- and Plate-Loaded Inverter

Again using the same tube parameters, it can be seen from Fig. 5 that

$$e_g = e - E_2$$

and from Fig. 6

$$i_p = \frac{\mu e_g}{r_p + r_1 + r_2}.$$

Then solving for E_2/E_1

$$\frac{E_2}{E_1} = \frac{r_2}{r_1}$$

and perfect balance is obtained when

$$r_2 = r_1.$$

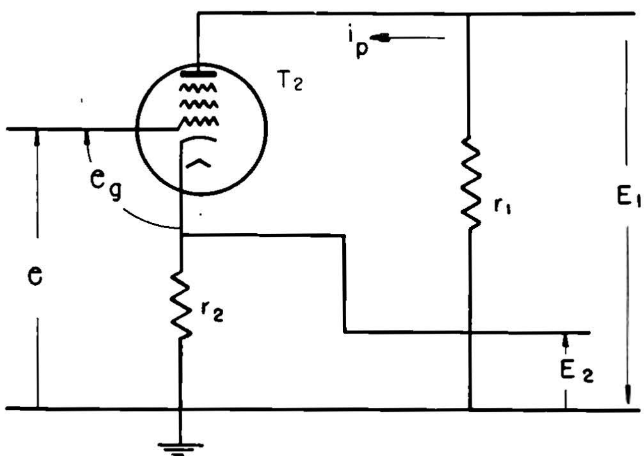

Fig. 5—Equivalent signal-voltage circuit of cathode- and plate-loaded inverter.

Define

$$N = \frac{E_2}{e}$$

and making

$$r_2 = r_1 = r$$

$$N = \frac{\mu r}{r_p + 2r + \mu r}. \quad (17)$$

The gain is essentially unity as seen from (17) since generally

$$\mu r >> r_p + 2r.$$

The conditions for balance are less rigid in this type of inverter, the only requirement being that the load resistors be equal. To compare the voltage gain of this single-tube circuit with the previous types, let us consider a voltage amplifier before the inverter as part of the voltage gain. This tube would have a gain

$$\frac{E}{e} = \frac{\mu r}{r_p + r}. \quad (18)$$

The inverter, having an approximate gain of unity per phase, will give an over-all gain, to a first approximation, equal to the gain of the first tube. This is also approximately the gain obtainable with the common-plate-loaded inverter, the exact ratios of the gains being easily obtainable but of little interest.

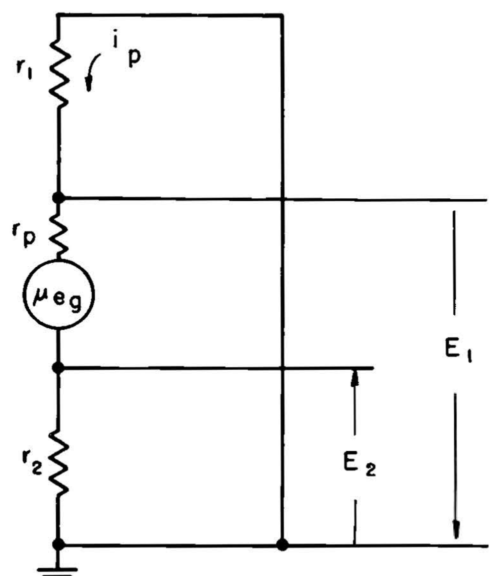

Fig. 6—Equivalent linear-tube-parameter circuit of cathode- and plate-loaded inverter.

Table I summarizes this data so that the three phase inverters may be compared.

TABLE I

Type	Balance Depends Upon	Exact Degree of Balance	Exact Inverter Gain	Approximate Inverter Gain	Approximate Relative Over-All Voltage Gain
Common Plate Load	$r_1 = r_2 = r$ $R_0 \approx r$ $N >> 3$	$\frac{N}{N+3}$	$N = \frac{\mu_2 r_2 - r_{p_2}}{r_{p_2} + r_2}$	$N \cong \frac{\mu_2 r_2}{r_{p_2} + r_2}$	1
Common Cathode Load	$r_1 = r_2 = r$ $R_0 \approx r$ $N >> 1$	$\frac{N}{N+1}$	$N = \frac{(\mu_2 + 1) r_2}{r_{p_2} + r_2}$	$N \cong \frac{\mu_2 r_2}{r_{p_2} + r}$	$\frac{1}{2}$
Plate-Loaded Cathode-Loaded	$r_1 = r_2 = r$	1	$N = \frac{\mu r}{r_p + 2r + \mu r}$	$N \cong 1$	1