DISCRIMINATOR

The correlation of electrical characteristics in resonant coupled circuits and staggered cascade circuits is demonstrated. From this result discriminator action is explained and conditions for best linearity established. Theoretical linearity curves are plotted

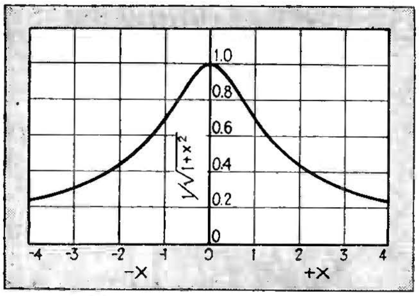

FIG. 1—Relative resonance characteristic showing absolute value of impedance as a function of Q and the ratio of frequency off resonance to the resonant frequency

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DISCRIMINATOR CIRCUITS are widely used to convert frequency-modulated signals to audio-frequency voltages. For this purpose it is desirable that the direct output vary linearly with the input frequency. This article discusses that linearity.

Logically we might write down the equations governing discriminator action without preliminary work. However, the relationships in a discriminator can be obtained as an extension of the problem of tuned coupled circuits. The response curves for tuned coupled circuits are, in turn, very closely related to those for simple tuned circuits. For these reasons it seems easiest to treat these three problems as a unit. In explaining discriminator action we shall therefore begin with a brief review of simple parallel resonance and of coupled circuits.

Review of Resonant and Coupled Circuits

The impedance of a resonant circuit of parallel R, L and C is given by:

$$\frac{1}{Z} = \frac{1}{R} + \frac{1}{j\omega L} + j\omega C \qquad (1)$$

It is customary to simplify this expression by normalizing the frequency variation in terms of the bandwidth of the tuned circuit between half-power points. On this basis the impedance becomes:

$$Z = \frac{R}{1 + jx} \qquad (2)$$

or

$$\left|\frac{Z}{R}\right| = \frac{1}{\sqrt{1 + x^2}} \qquad (3)$$

where

$x \cong 2\,Q\,\Delta f\,/\,f_0$

Δf = frequency off resonance

Figure 1 is a plot of the well-known normalized resonance curve.

To get a physical picture of what takes place in coupled circuits we can think of the response curve as the product of two resonance curves staggered in frequency a certain number of bandwidths.

For example, two resonant circuits can be regarded as the coupling impedances in a two-stage amplifier as indicated in Fig. 2.

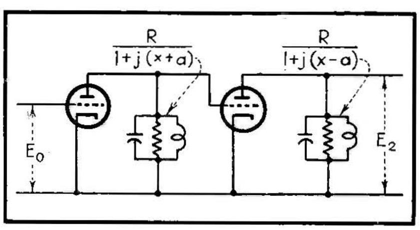

FIG. 2—Staggered resonant circuits used as coupling elements in cascade amplifier

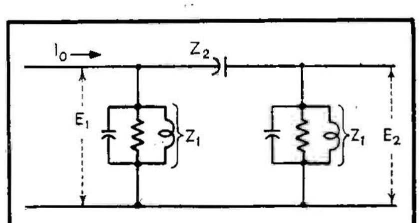

FIG. 3—Identically tuned coupled-resonant circuits could be used as a coupling element

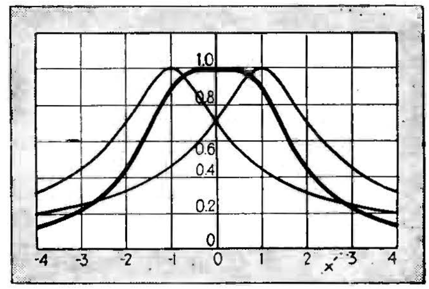

FIG. 4—Symmetrically detuned resonant circuits combine to give critical flatness at $a = 1$. The ordinates of the product curve are multiplied by two to improve readability

The voltage amplification is given by:

$$\frac{E_2}{E_0} = g_m{}^2R^2 \frac{1}{[1 + j\,(x + a)]\,[1 + j\,(x - a)]} \qquad (4)$$

or

$$\left|\frac{E_2}{E_0}\right| = \frac{g_m{}^2\,R^2}{\sqrt{[1 + (x + a)^2]\,[1 + (x - a)^2]}} \qquad (5)$$

Here each circuit is resonated a half-bandwidths from the mean frequency.

In the case of the coupled circuits shown in Fig. 3, we try to obtain an analogous expression for the transfer impedance, E_2/I_0. It will be worth working this out in detail because the results are directly applicable to the discriminator. In Fig. 3 we have:

$$E_1 = \frac{Z_1\,(Z_1 + Z_2)}{2Z_1 + Z_2}\,I_0 \qquad (6)$$

and

$$E_2 = \frac{Z_1}{Z_1 + Z_2}\,E_1 \qquad (7)$$

Combining Eq. (6) and (7) we have:

$$\frac{E_2}{I_0} = \frac{Z_1{}^2}{2Z_1 + Z_2} \qquad (8)$$

The impedance Z_2 is that of a physical capacitor. It is very convenient, however, to neglect the variation of this capacitance over the small percentage range of frequencies considered in the resonance curves. This enables us to simplify Z_2 to

$$Z_2 = -\,jkR. \qquad (9)$$

LINEARITY

Substituting Eq. (2) and (9) in Eq. (8), we obtain

$$\frac{E_2}{I_0} = \frac{[R/(1+jx)]^2}{[2R/(1+jx)] - jkR} \quad (10)$$

or

$$\frac{E_2}{I_0R} = \frac{1}{(2 - jk + kx)(1 + jx)} \quad (11)$$

Comparison of Symmetrically Detuned Cascaded Circuits with Identically Resonated Coupled Circuits

It would be desirable to reduce Eq. (11) to a form similar to that of Eq. (4). In order to do this it is clearly necessary to get the coefficient of x in the second factor of Eq. (11) changed from k to j. This is accomplished by multiplying numerator and denominator by j/k:

$$\frac{E_2}{I_0R} = \frac{j/k}{[1 + j(x + 2/k)][1 + jx]} \quad (12)$$

If we measure normalized frequency deviations from $1/k$ instead of from zero by changing the variable, so that $x + 1/K = y$, Eq. (12) becomes

$$\frac{E_2}{I_0R} = \frac{j/k}{[1 + j(y + 1/k)][1 + j(y - 1/k)]} \quad (13)$$

Except for a multiplying factor and a shift of abscissas this is identical to Eq. (4), the equation for staggered circuits. In Eq. (13) $1/k$ is to be identified with a, the half bandwidths off resonance, of Eq. (4).

The choice of direct capacitive coupling rather than mutual inductive coupling was made to simplify the computation. If inductive coupling had been used the results would have been the same except that no shift in abscissas would have occurred.

The relationship between staggered and resonant circuits provides the physical interpretation of the resultant curves. The product of two widely-spaced curves gives a double resonant peak while the product of two coincident peaks yields a single sharp peak. Thus we are led to investigate the possibility of choosing the spacing between peaks in such a manner as to make the two peaks coalesce to give a critically-flat curve.

One way of determining the conditions of this consists of differentiating Eq. (5) with respect to x and determining the value of a that would make the two peaks just meet. A second method consists of setting the second derivative of the curve equal to zero at $x = 0$ and solving for a. As is well-known, these conditions lead to the choice of $a = 1$ for critical flatness. Curves showing the two resonant curves and their product for this condition are shown in Fig. 4.

Discriminators

We found it convenient to obtain a physical picture of coupled circuits by means of cascaded staggered circuits. In just the same way it is useful to consider a very simple discriminator circuit before considering the general case. Figure 5 shows an idealized discriminator circuit of this sort.

Detectors are arranged to provide two opposing direct voltages one of which is proportional to the magnitude of the impedance of one circuit while the other is proportional to the impedance of the second circuit. As before the resonant frequency of each circuit is staggered from a mean value by a half-bandwidths. Thus we have

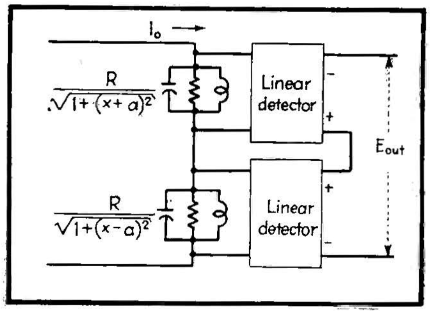

FIG. 5—Simple staggered resonant-circuit discriminator

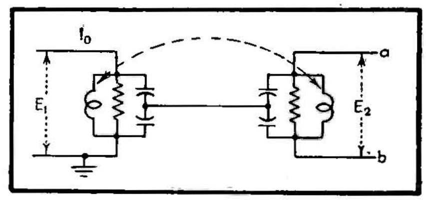

FIG. 6—Basic coupled resonant-circuit discriminator

$$\left|\frac{E_{out}}{I_0R\sqrt{2}}\right| = \frac{1}{\sqrt{1 + (x - a)^2}} - \frac{1}{\sqrt{1 + (x + a)^2}} \quad (14)$$

It has been pointed out by Travis[1] that the more conventional type of discriminator has a characteristic expressible in this form. Let us consider the circuit shown in Fig. 6. From our previous analysis, we see that while the frequency discrimination of the circuit of Fig. 5 corresponds to that of the circuit in Fig. 2, the discriminator of Fig. 6 behaves like the coupled circuits of Fig. 3.

The voltages E_a and E_b in Fig. 6 are fed to separate linear rectifiers whose outputs are opposed. It is assumed that the coils in Fig. 6 are

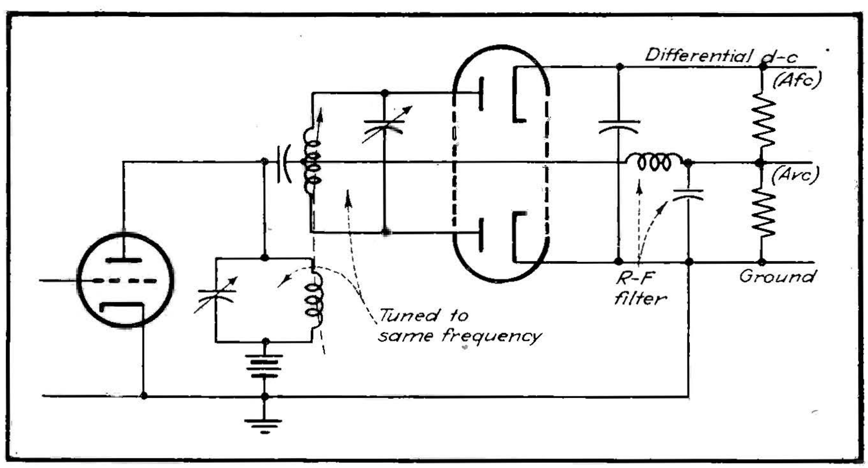

FIG. 7—Practical discriminator circuit which has the same action as the basic coupled resonant circuit of Fig. 6

inductively coupled. If negligible load current is drawn from the points a and b the voltage E_a is made up of half the difference between the primary and secondary voltages. Similarly the voltage E_b is made up of half the sum of these two voltages. Whether the primary and secondary circuits are inductively coupled or capacitively coupled makes no essential difference. Hence except for a shift in the variable and a constant factor, the primary and secondary circuits of Fig. 6 are related in the same way that the corresponding circuits in Fig. 3 are related. Thus we are led to consider the magnitudes $|E_1 + E_2|$ and $|E_1 - E_2|$ in Fig. 3. Combining Eq. (6) and (8) we write:

$$\frac{E_1 + E_2}{I_0} = \frac{Z_1 (Z_1 + Z_2) + Z_1^2}{2Z_1 + Z_2} = Z_1 \quad (15)$$

and

$$\frac{E_1 - E_2}{I_0} = \frac{Z_1 (Z_1 + Z_2) - Z_1^2}{2Z_1 + Z_2} = \frac{Z_1 Z_2}{2Z_1 + Z_2} \quad (16)$$

Substituting the equivalent expressions for the impedances, we have

$$\frac{E_1 + E_2}{I_0 R} = \frac{1}{1 + jx} \quad (17)$$

and

$$\frac{E_1 - E_2}{I_0 R} = \frac{\frac{1}{1 + jx}(-jk)}{\frac{2}{1 + jx} - jk} = \frac{-jk}{2 - jk + kx} \quad (18)$$

Forming the difference between the magnitudes of Eq. (17) and (18) we write

$$\frac{|E_1 + E_2| - |E_1 - E_2|}{|I_0| R} = \frac{1}{\sqrt{1 + x^2}} - \frac{1}{\sqrt{1 + (x + 2/k)^2}} \quad (19)$$

Making the substitution $y = x + 1/k$ as before and writing $h(y)$ for the value of the function we obtain

$$h(y) = \frac{1}{\sqrt{1 + (y - a)^2}} - \frac{1}{\sqrt{1 + (y + a)^2}} \quad (20)$$

where $a = 1/k$.

This has the same form as Eq. (14).

The practical discriminator shown in Fig. 7[2] can be made the same as that shown in Fig. 6 provided that proper turns ratios are used between the primary and secondary windings and provided the two windings have the same Q. The two rectifier outputs are proportional to $|E_b|$ and $|E_a|$.

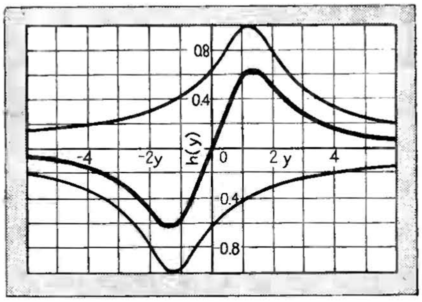

FIG. 8—Discriminator characteristics which result from a coupling equivalent to a frequency spacing between circuit resonances which corresponds to $a = \sqrt{3/2}$

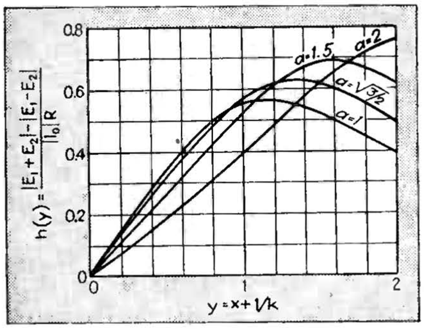

FIG. 9—Discriminator curves for various values of a

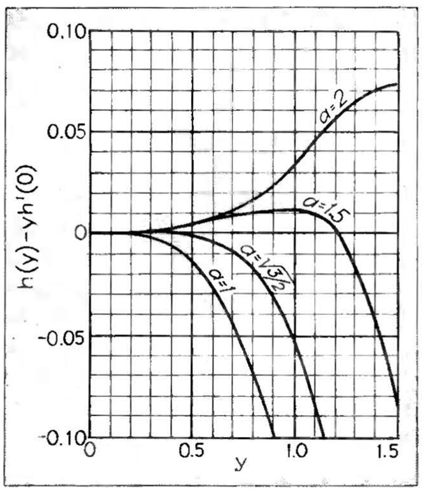

FIG. 10—Deviations of discriminator curves from tangents at their cross-over point

The symmetrical conditions assumed in Fig. 2 in deriving coupled-circuit relationships are by no means necessary but are mathematically convenient. Similarly it is likely that symmetrical conditions are not necessary in the case of discriminators. Nevertheless the assumption of such conditions is very helpful in carrying out the arithmetic in each case. The writer has not tried to get a solution of unsymmetrical cases.

Condition for Best Linearity

Let us investigate the shapes of the curves given by Eq. (20). Clearly if we let the two generating resonance curves be spaced by too great a frequency the resultant curve will be badly nonlinear. Similarly the curve for an opposite extreme will have a reverse curvature and will be just as undesirable. If we investigate the linearity at the center by considering the derivatives at the point $y = 0$ we find,

$$h^{I}(0) = \frac{2a}{(1 + a^2)^{-3/2}}$$

$$h^{II}(0) = 0$$

$$h^{III}(0) = \frac{6a\,(2a^2 - 3)}{(1 + a^2)^{7/2}}$$

$$h^{IV}(0) = 0$$

$$h^{V}(0) = \frac{30a\,(8a^4 - 40a^2 + 15)}{(1 + a^2)^{11/2}}$$

$$h^{VI}(0) = 0$$

$$h^{VII}(0) = \frac{630a\,(226a^6 - 78a^4 + 90a^2 - 35)}{(1 + a^2)^{13/2}}$$

It will be noticed that for $a = \sqrt{3/2}$ the second, third, and fourth derivatives are all zero, making for exceptionally good linearity.

Figure 8 shows the discriminator characteristic for this critical case where the third derivative vanishes. Figure 9 shows a family of curves for various values of a between 1 and 2. For convenience the lower halves of these curves have been omitted.

It would have been useful to permit the frequency deviation y to vary sinusoidally and to expand the resultant curve for $h(y)$ as a Fourier series. This would permit the computation of distortion factors for various values of a and frequency swing. Unfortunately lack of time has prevented the completion of this numerical work. However, some estimates of the distortion can be obtained from a study of the curves in Fig. 10. These curves show the difference between the discriminator curves of Fig. 9 and tangents drawn to them at the origin. It will be noticed that the curve for $a = \sqrt{3/2}$ differs from its tangent by less than 0.2 percent for values of y less than 0.5. However, the curve for $a = 1.5$ lies close to its tangent over a considerably larger interval.

References

(1) Travis, Charles, Automatic Frequency Control, *Proc. IRE*, 23, Oct. 1935, p. 1125.
(2) Foster, D. E. and Seely, S. W., Automatic Tuning, Simplified Circuits, and Design Practice, *Proc. IRE*, 25, Mar. 1937, p. 289.