Modulation Limits in F-M

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Since the generation of adjacent channel interference in f-m systems depends on the amplitude as well as the frequency of the modulating signal, such interference may be minimized by restricting the modulation in either of two ways. The authors show the permissible frequency deviations, for specified degrees of adjacent channel interference, at various audio frequencies, and in general make clear the notion of overmodulation as it applies to f-m systems

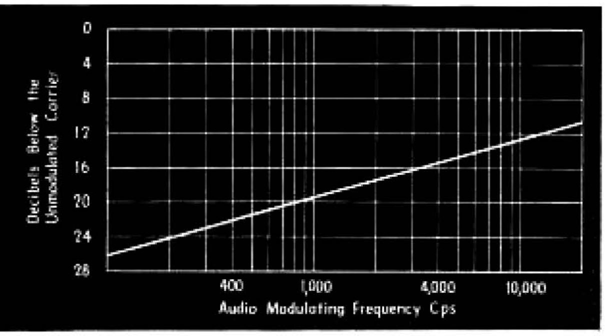

Fig. 1—The amplitude of the sideband at the edge of the band relative to that of the unmodulated carrier, for a frequency deviation of 75 kc. The higher the audio frequency, the stronger the sideband

In an amplitude-modulated system, the degree of modulation is expressed as the percentage variation of the carrier amplitude. If this variation exceeds 100 per cent, distortion of the signal program is introduced. By properly limiting the frequency range of the modulating signal, the radiated frequencies can be kept within a specified band. On the other hand, in a frequency-modulated system, in order to confine the radiated frequencies to a specified band it is necessary to limit properly *both* the intensity *and* the frequency range of the modulating signal. This fundamental difference between a-m and f-m in the general problem of interference is evident from a comparison of the sidebands of the two systems. In an amplitude-modulated wave a modulating signal of 10,000 cps produces only two side frequencies, each differing from the carrier by 10,000 cps. This same signal used to frequency-modulate a carrier introduces an infinite number of side frequencies which differ from the carrier frequency by integral multiples of 10,000 cps. The magnitudes of the higher order side frequencies can be kept small by properly limiting the degree of modulation. A consideration of these factors for a frequency-modulated system is the subject of this paper.

The instantaneous frequency, ω, of a frequency modulated wave is defined by the equation

$$\omega = \omega_c (1 + k \cos \omega_v t) \qquad (1)$$

$\omega_c = 2\pi f_c$ = unmodulated carrier frequency in radians per sec.

$\omega_v = 2\pi f_v$ = modulating signal frequency in radians per second.

k = a factor proportional to the intensity of the modulating signal and independent of the modulating frequency.

The expression for the instantaneous radio frequency voltage e is given by the expression

$$e = E_c \sin \int \omega dt$$

$$= E_c \sin \left(\omega_c t + \frac{k\omega_c}{\omega_v} \sin \omega_v t\right)$$

$$= E_c \sin (\omega_c t + m \sin \omega_v t) \qquad (2)$$

where

E_c = amplitude of the unmodulated carrier

$m = \dfrac{k\omega_c}{\omega_v}$ = modulation index

The modulation index, m, is proportional to the intensity of the modulating signal k and inversely proportional to the frequency of modulation ω_v. By expansion, Equation (2) can be expressed in the form

$$e = E_c \Big\{ J_0 (m) \sin \omega_c t \quad \text{(carrier)}$$
$$- J_1 (m) \sin (\omega_c - \omega_v) t$$
$$\cdots\cdots$$
$$+ (-1)^n J_n (m) \sin (\omega_c - n\omega_v) t$$
$$\text{(lower side frequencies)}$$
$$\cdots\cdots$$
$$+ J_1 (m) \sin (\omega_c + \omega_v) t$$
$$\cdots\cdots$$
$$+ J_n (m) \sin (\omega_c + n\omega_v) t \Big\}$$
$$\text{(upper side frequencies)} \qquad (3)$$

where

$J_n (m)$ = A Bessel function of the quantity m and of order n.

$n = 0, 1, 2, 3 \ldots\ldots$

Equation (3) shows an infinite number of side frequencies extending in both directions from the carrier frequency. The ratio of the amplitude of the nth side frequency to the amplitude of the unmodulated carrier is $J_n(m)$. The amplitudes of the side frequencies extending beyond a specified frequency band will be small if the modulation index, m, has a small value. The value of m can be made as small as desired by limiting the intensity of the modulating signal.

The Federal Communications Commission has tentatively specified[1] a "maximum bandwidth of emission of 200 kc". Limitations on the strength of signals on frequencies beyond this band width are not specified.

A Practical Example

Previous treatments of frequency modulation have for the most part considered the permissible frequency deviation, kf_c, at maximum modulation to be equal to one-half the allowable bandwidth. Under these conditions, the magnitudes of the side frequencies beyond the specified band may be sufficient to cause serious interference in adjacent channels. As an illustration of this, a bandwidth of 150 kc and a modulating frequency of 15 kc will be assumed. If the deviation at maximum modulation is made equal to one-half the bandwidth, then

$$kf_c = \Delta f_c$$
$$= 75{,}000 \text{ cps.}$$

and

$$m = \frac{kf_c}{f_v} = 5$$

where Δf_c = one-half the allowable bandwidth.

At the edge of the band, n is equal to 5. The ratio of the amplitude of this side frequency to that of the unmodulated carrier is 0.261 or —11.7 db. A signal of this magnitude may be sufficient to cause interference in an adjacent channel. For lower modulating frequencies Fig. 1 shows the magnitude of the side frequencies at the edge of a one-half bandwidth of 75 kc in db below the unmodulated carrier plotted as a function of the signal frequency.

The amplitude at the edge of the frequency band can be reduced to any desired level by adequately restricting the frequency deviation, kf_c, of the carrier. If the frequency band is limited by filters in the radio frequency stages, instead of by restricting the frequency deviation, distortion will be present in the reproduced signal.

Figure 2 is a family of curves showing the ratio

$$\frac{\text{maximum frequency deviation}}{\text{one-half bandwidth}}$$

plotted as a function of the ratio

$$\frac{\text{one-half bandwidth}}{\text{audio signal frequency}}$$

These curves are for the condition that the amplitude at the edge of the frequency band shall be a specified number of decibels below the unmodulated carrier. The levels are indicated on the curves. These values were obtained from cross plots of Bessel functions that are tabulated[2] for values of m up to 29. For values of m from 29 to 500 the functions were computed from a relation given by Nicholson[2]. The curves, when plotted in this form, are quite universal and cover a range that should prove sufficient for any required calculations.

Use of Curves in Fig. 2

To illustrate the use of these curves, it will be assumed that the amplitude of frequencies outside a bandwidth of 150 kc ($\Delta f_c = 75$ kc) must be at least 60 db below the level of the unmodulated carrier. At the edge of the band, the value of n is determined from the relation

$$n = \frac{\Delta f_c}{f_s}$$

and since

$$m = \frac{kf_c}{f_s}$$

then

$$\frac{m}{n} = \frac{kf_c}{\Delta f_c} = \frac{\text{frequency deviation}}{\text{one-half bandwidth}}$$

For an audio modulating signal of 7,500 cps, the value of n is equal to 10. Referring to the 60 db curve of Fig. 2; the maximum permissible frequency deviation is seen to be 0.48 times the one-half bandwidth or 36 kc. The maximum frequency deviation depends upon the frequency of the audio signal and its smallest value corresponds to the highest frequency of modulation. As a result of this, it might naturally be thought that the maximum deviation of the carrier frequency would be set by the highest audio frequency. This is not actually the case, however, because in the usual broadcast program the signal intensities at the high frequency end are of relatively low level.

Examination of the relative intensity-frequency distribution in speech and music brings out the fact that the audio frequencies in the vicinity of 400 cps are the ones most likely to produce adjacent channel interference. This is illustrated in Fig. 3. Curves 1 and 2 show the relative peak pressures, expressed in db below the maximum, of conversational speech for men and women respectively. These two curves were drawn from the data of Dunn and White[3]. The relative distribution of peak power[4] for a 75 piece orchestra is shown by curve 3. Curve 4 is a plot of the ratio

$$\frac{\text{maximum frequency deviation}}{\text{one-half bandwidth}}$$

expresed in db relative to the ratio at 400 cps. In determining this latter curve, a bandwidth of 150 kc has been assumed and the intensities outside of this band are limited to 60 db or more below the unmodulated carrier.

These curves indicate that if the permissible deviation at 400 cps is not exceeded, the modulation at the higher and lower audio frequencies will fall within the required limits, that is, the frequency deviation will always be less than the limit set by curve 4. At 400 cps the maximum carrier frequency deviation is 0.924 times the one-half bandwidth (Fig. 2). From these results, it can be concluded that if the peak audio intensities for speech or music in the vicinity of 400 cps produce a frequency deviation of less than 90

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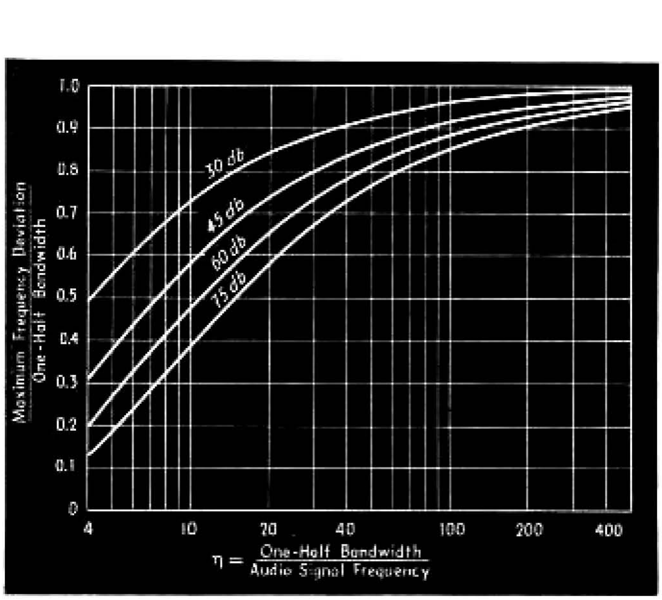

Fig. 2—Universal curves showing the permissible frequency deviation for various audio frequencies

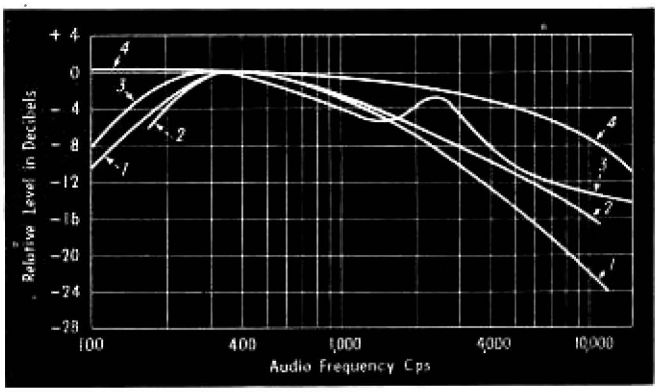

Fig. 3—The relative energy of various program types as functions of frequency: 1, male speech; 2, female speech; 3, 75-piece orchestra; 4, permissible deviation relative to 400 cps

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per cent of the one-half bandwidth, frequencies outside this band will be at least 60 db below the unmodulated carrier. It is necessary to qualify this conclusion when certain percussion instruments such as the triangle and cymbals are present in the program. With these instruments intense peaks occasionally occur[5] at frequencies as high as 10,000 cps. Under these conditions it may be necessary to limit the maximum frequency deviation to 50 per cent of the one-half bandwidth.

It should be pointed out that a small increase in the ratio of the maximum carrier deviation to the one-half bandwidth may result in a large increase of the magnitudes of frequencies outside of the assigned band. For example, an increase of this ratio of from 0.90 to 0.95 results in an increase in the level at the edge of the band of from —75 db to —45 db for a signal frequency of 400 cps.

The principles outlined in this paper appear to offer a reasonable basis for the establishment of modulation limits in a frequency-modulated system. The ratio

$$\frac{\text{maximum frequency deviation}}{\text{one-half bandwidth}}$$

is a convenient means of expressing the maximum permissible degree of modulation. If this limit is properly set, interference in adjacent bands may be avoided.

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