the amplifier, and the output of the amplifier is viewed on an oscilloscope screen. The frequency of the pulse generator may be varied to determine the width of the pass band of the amplifier and the type of distortion present at the edges of the pass band. For example, a simple resistance-capacitance coupled amplifier at low frequencies acts much like a series resistance-capacitance circuit with the output voltage appearing across the resistance. The resulting wave forms are similar then to those of Fig. 2, and the approximate lower half-power frequency would be that when the wave form for $\theta=(1/2\pi)$ of Fig. 12(a) would appear. The upper half-power frequency may be obtained in a similar manner, and the wave form to be used is that of Fig. 12(b) for $\theta=2\pi$. Resonance occurring in the amplifier may be found by noticing for what frequencies of the pulse generator the output voltage becomes very large. The output voltage wave forms should be similar to those of Fig. 9. When the sinusoidal or nearly sinusoidal wave form is that for $m=2\pi$, the frequency of the pulse generator is the same as the resonant frequency of the amplifier.

It has been shown[10] that if the steady-state response of an amplifier is known to a saw-tooth wave of period T, the steady-state response to any nonsinusoidal wave of the same period T may be calculated. The same is true if a pulse wave is used. For example, if $e_0(t)$ is the steady-state output response voltage of a pulse of period T applied to an amplifier, and if $e(t)$ is any other nonsinusoidal wave of period T, then the steady-state response e_s of the amplifier to $e(t)$ is

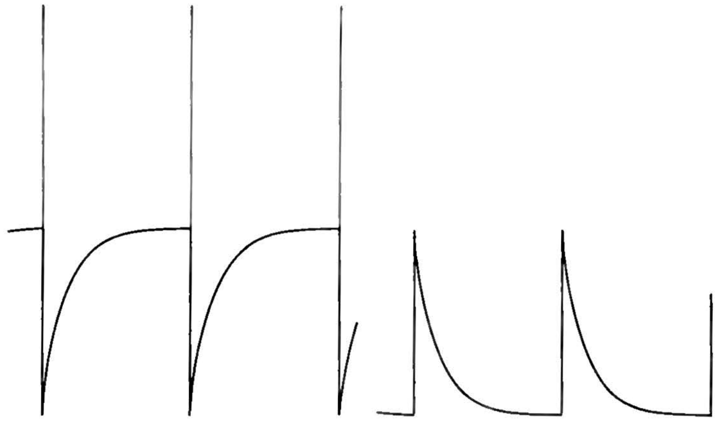

Fig. 12—Calculated wave forms for (a) the lower, (b) the upper half-power frequencies.

$$e_s = \int_0^T e(t-\tau)e_0(\tau)d\tau, \tag{21}$$

or another equivalent form is

$$e_s = \int_{t-T}^{t} e(\tau)e_0(t-\tau)d\tau. \tag{22}$$

If the equations for e and e_0 are known, it is possible to integrate (21) or (22). If equations are not known for either e or e_0 or both, a numerical solution is still possible as outlined in the previous reference.[10]

Diode Phase-Discriminators*

R. H. DISHINGTON†

***Summary*—Two sinusoidal phase-discriminators are analyzed and it is found that universal curves of their general phase characteristics can be plotted as a function of two parameters. From these curves it is concluded that the resistances in series with the tubes and also the tube resistances themselves are the most important factors in determining optimum performance.**

Introduction

THE PHASE-DISCRIMINATOR, otherwise known as phase-comparator or phase-detector gives a measurement of the phase difference between two waves. Diode discriminators, having the advantage of simplicity, indicate the phase angle by a voltage at the output terminals. At present, the principles of operation are well known, but there is a noticeable lack of an accurate analysis of the circuits.[1,2] The present paper deals with the problem by applying a recently introduced general method of diode circuit analysis.[3] For all practical purposes, this gives an exact solution. The circuits' general characteristics are given graphically, and only a simple calculation must be made to obtain the complete phase-characteristic for any practical values of the circuit parameters.

The Basic Method

In footnote reference 3 it was shown that a tube and any series resistance R_s have a *combination characteristic*

$$i_b = Ke_d^{\alpha_c} \tag{1}$$

where i_b is the plate current, e_d is the voltage across both the tube and R_s, and K and α_c are constants. Mathematically,

$$e_d = e_b + i_bR_s \tag{2}$$

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[1] W. L. Emery, "Ultra-High-Frequency Radio Engineering," Macmillan Co., New York, N. Y., 1944; p. 41.

[2] L. I. Farren, "Phase detectors, some theoretical and practical aspects," *Wireless Eng.*, vol. 23, pp. 330–340; December, 1946.

[3] R. H. Dishington, "Diode circuit analysis," *Elec. Eng.*, vol. 67, pp. 1043–1049; November, 1948.

where e_b is the plate voltage of the tube. To use the solutions presented further on, two quantities must be computed. First, referring to Figs. 1 and 5,

$$i_2 = \frac{e_{1\,max}}{R_{bb}} \quad (3)$$

and, from (2),

$$E_{21} = e_b\big]_{i_2} + i_2 R_s \quad (4)$$

where $e_b]_{i_2}$ is the plate voltage of the tube at i_2, taken directly off the static plate characteristic. Second, the exponent α_c can be found very simply, as explained in footnote reference 3.

The Simple Sinusoidal Phase-Discriminator

Phase difference between two sinusoidal waves can be measured by the circuit in Fig. 1. The magnitudes of the *open circuit* input voltages e_x and e_y are assumed

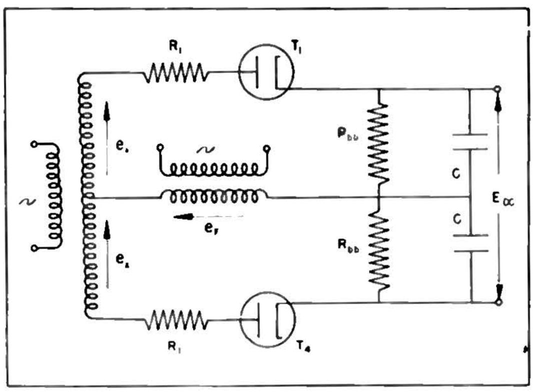

Fig. 1—The simple phase-discriminator.

equal. Both sides of the circuit are identical except for the net voltage applied to each. The driving voltages for T_1 and T_4 respectively are

$$\left.\begin{aligned} e_1 &= e_y + e_x \\ e_4 &= e_y - e_x \end{aligned}\right\}. \quad (5)$$

Adding a fixed lead of $\pi/2$ to e_y, to resolve the ambiguity in ϕ for positive and negative angles,

$$\left.\begin{aligned} e_1 &= E \sin\left(\omega t + \phi + \frac{\pi}{2}\right) + E \sin \omega t \\ e_4 &= E \sin\left(\omega t + \phi + \frac{\pi}{2}\right) - E \sin \omega t \end{aligned}\right\} \quad (6)$$

Transforming (6),

$$\left.\begin{aligned} e_1 &= (e_{1\,max}) \sin\left(\omega t + \phi + \frac{\pi}{4}\right) \\ e_4 &= (e_{4\,max}) \cos\left(\omega t + \phi + \frac{\pi}{4}\right) \end{aligned}\right\} \quad (7)$$

where

$$\left.\begin{aligned} e_{1\,max} &= 2E \cos\left(\frac{\phi}{2} + \frac{\pi}{4}\right) \\ e_{4\,max} &= 2E \sin\left(\frac{\phi}{2} + \frac{\pi}{4}\right) \end{aligned}\right\}. \quad (8)$$

The peak values of e_1 and e_4 are functions of ϕ, but not of time.

Equation (7) reveals that the voltages applied to the two opposite sides of the circuit are always 90° out of phase. This means that, except for a short period of overlap, one tube conducts while the other does not. Little error is introduced if both halves are assumed completely independent. Once this assumption is made, reduction to the equivalent circuit is simple, each half of the circuit being reduced separately. The calculated output of T_4 is then subtracted from that of T_1 to give E_{DC} (Fig. 1). Completely general curves for the solution are shown in Figs. 2, 3, and 4. To use the curves, it is necessary to evaluate R_s. In the present circuit, R_s is the sum of the internal resistances of e_x and e_y plus R_1. The curves are plotted for various values of the ratio $(E_{21}/e_{1\,max})_{-90°}$ at $\phi = -90°$. Actually $E_{21}/e_{1\,max}$ changes with ϕ. A correction for this is used to obtain the solution. The results give E_{DC}/E for negative values of ϕ, but the positive angles give the same shape of characteristic with negative voltage.

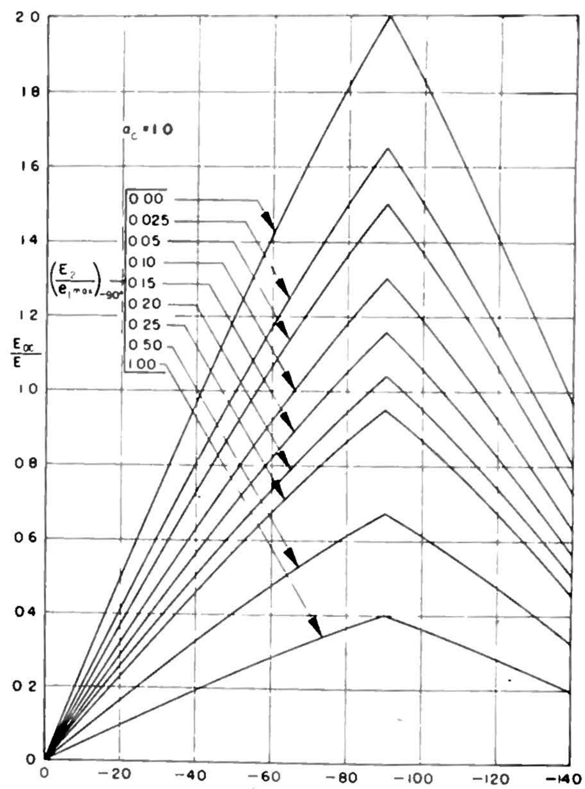

Fig. 2—General phase characteristics of the simple discriminator with sinusoidal input when $\alpha_c = 1.0$. To find $(E_{21}/e_{1\,max})$ use the one value $e_{1\,max} = 2E$.

The sensitivity of phase measurement for any given value of α_c is a function of $E_{21}/e_{1\,max}$, which can be expressed

$$\frac{E_{21}}{e_{1\,max}} = \frac{1}{R_{bb}}\,\frac{e_b}{i_2}\bigg]_{i_2} + \frac{R_s}{R_{bb}}. \quad (9)$$

Equation (9) makes it apparent that large values of R_{bb} and small values of R_s tend to lower $E_{21}/e_{1\,max}$ and thereby increase the sensitivity. The quantity $e_b/i_2]_{i_2}$ is of the order of magnitude of R_{T_1}, so a low R_{T_1} also in-

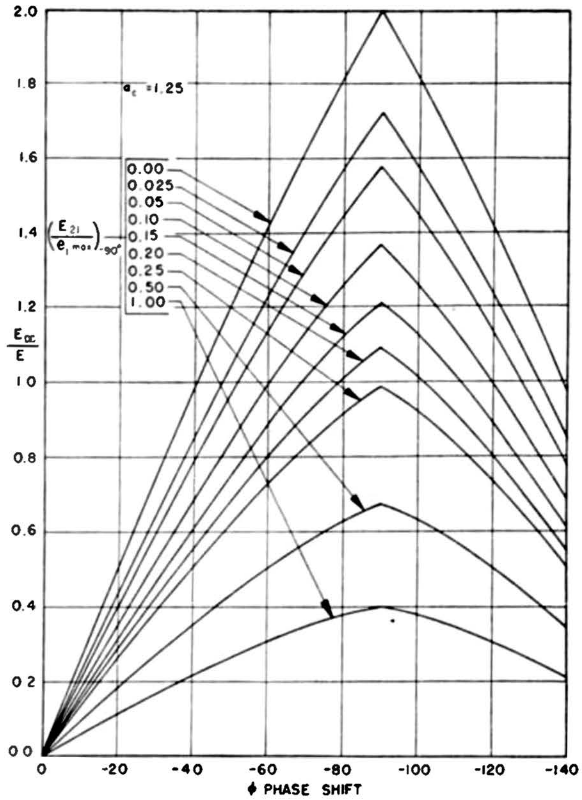

Fig. 3—General phase characteristics of the simple discriminator with sinusoidal input when $\alpha_c = 1.25$. To find $(E_{21}/e_{1\,max})$ use the one value $e_{1\,max} = 2E$.

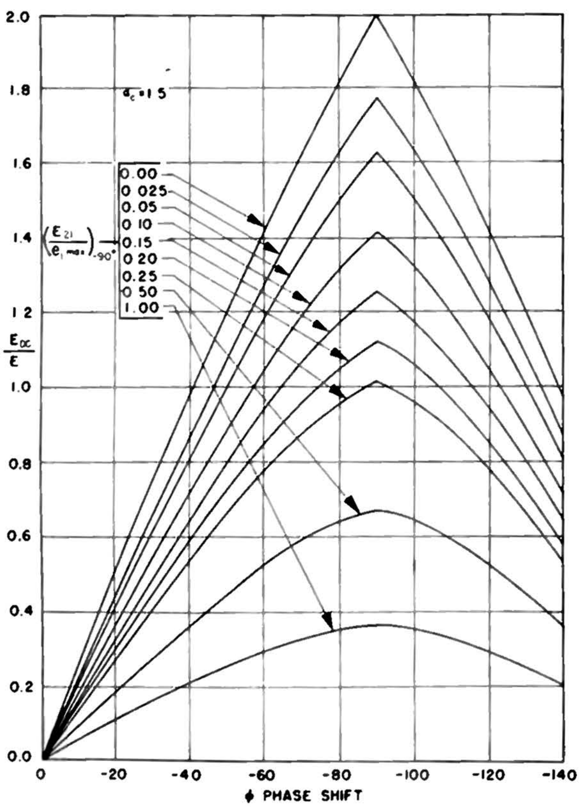

Fig. 4—General phase characteristics of the simple discriminator with sinusoidal input when $\alpha_c = 1.5$. To find $(E_{21}/e_{1\,max})$ use the one value $e_{1\,max} = 2E$.

creases the sensitivity. (For values of R_{T_2} see footnote reference 3.)

The linearity is better for values of α_c near 1.0. However, unless the tube α_s is originally near unity, α_c can only be made linear by adding R_1. From the foregoing, this increases R_s and decreases the sensitivity. For high sensitivity, the difference in nonlinearity of the output between $\alpha_c = 1.0$ and $\alpha_c = 1.5$ is very small. Therefore, an optimum design will have no R_1, making R_s as small as possible.

The Balanced Sinusoidal Phase Discriminator

Another well-known comparator is the balanced circuit shown in Fig. 5. The tubes and resistors R_1 are the same for each branch. Both RC loads are also similar. Given the same conditions for e_x and e_y, the driving

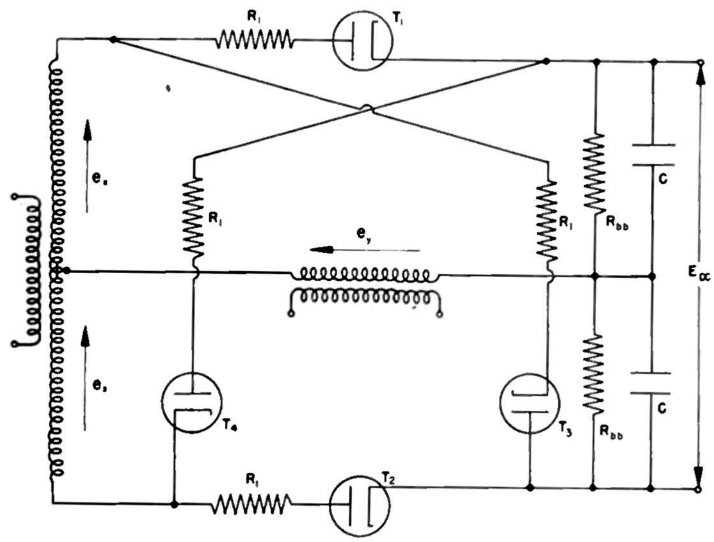

Fig. 5—The balanced phase-discriminator.

voltages for tubes T_1, T_2, T_3, and T_4 are e_1, e_4, $-e_1$, and $-e_4$ respectively. Again, except for a slight overlap, each tube conducts when the other three do not. Consequently, it is assumed that the two halves of the circuit are separable. It is conventional to show E_{bb} as positive with respect to the reference diode plate. Tube T_1 is chosen as the reference for the top half, and inasmuch as the constant output voltage is actually produced across an RC load, E_{bb} will be negative for negative phase angles. For the same conditions, E_{bb} tends to make the plate of T_4 positive. For this reason, it is important in the derivation to remember that for negative ϕ, T_1 operates class C and T_4 operates class AB or A. The second half of the circuit produces an output indentical to the first and in series with it. Thus, the two output voltages are added to give the total E_{DC}. The final solution for sinusoidal input voltages is given in Figs. 6, 7, and 8. Remarks on how to calculate $(E_{21}/e_{1max})_{-90°}$ are exactly the same for this circuit as for the simple comparator. Also the effects of the various resistors on the sensitivity are the same as before. Examining the curves, it appears that unless the flat-topped phase characteristic of Fig. 6 is desired for some particular reason, better sensitivity with more over-all performance is obtained with operation as near to $\alpha_c = 1.5$ as possible. This means less R_1; but a precaution is necessary here. Originally, it was assumed that E_{bb} had a constant value for each ϕ. This is made

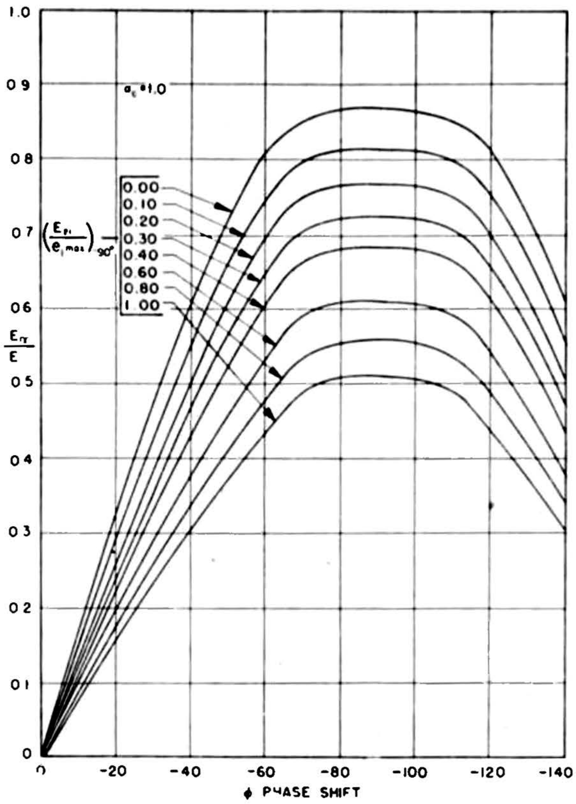

Fig. 6—General phase characteristics of the balanced discriminator with sinusoidal input when $\alpha_c = 1.0$. To find $(E_{21}/e_{1\,max})$ use the one value $e_{1\,max} = 2E$.

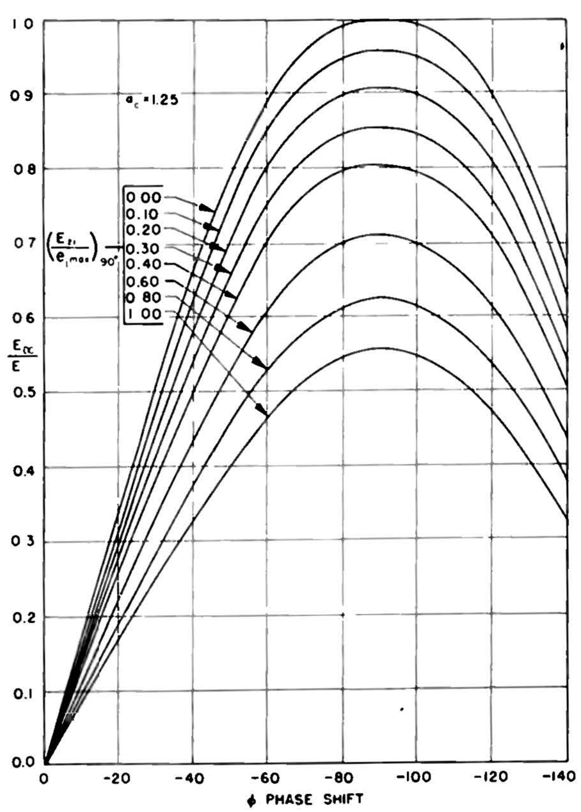

Fig. 7—General phase characteristics of the balanced discriminator with sinusoidal input when $\alpha_c = 1.25$. To find $(E_{21}/e_{1\,max})$ use the one value $e_{1\,max} = 2E$.

possible by a large enough time constant $R_{bb}C$. Now, however, the capacitor can discharge through T_4 for example, and unless $(R_s + R_7)C$ is large, E_{bb} may not remain constant. This generally means that some R_1 must be added.

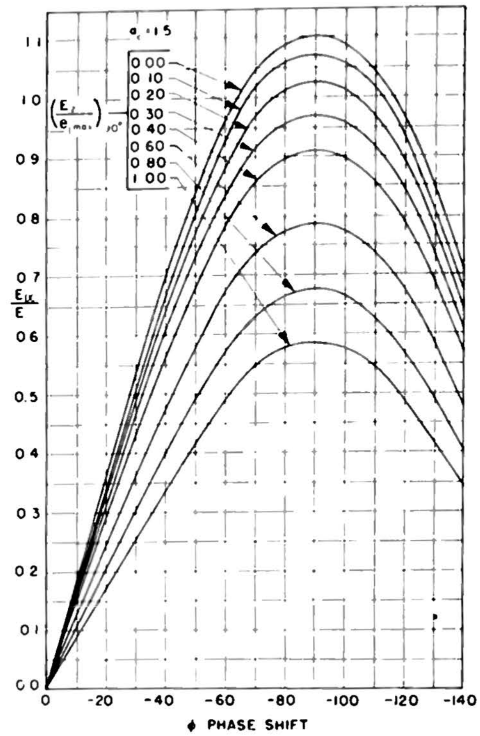

Fig. 8—General phase characteristics of the balanced discriminator with sinusoidal input when $\alpha_c = 1.5$. To find $(E_{21}/e_{1\,max})$ use the one value $e_{1\,max} = 2E$.

Conclusions

The total phase characteristics of two basic types of phase-discriminators are given in a form which enables quick calculation of the proper performance curve. Only two assumptions are made; one, that the ripple across the load is negligibly small; and two, that each tube conducts when the others are nonconducting. The first assumption is easily justified, and the second introduces only a minute error in practical cases. It appears that, in both circuits, the sensitivity is increased by large R_{bb}, and small R_1 and tube resistance. However, the value of R_1 must be large enough in the balanced circuit to ensure the constancy of E_{bb} by giving a large time constant $(R_s + R_l)C$. The slight increase in linearity, over only a part of the range, which is gained by adding R_1 is more than offset by the undesirable loss of sensitivity.

The balanced circuit seems to be less desirable than the simple one, but there is one important feature to consider. The output of the simple circuit is the difference between two large voltages. This gives inaccurate operation for small phase angles in a practical circuit where tubes and resistors are not perfectly matched. To its advantage, the balanced circuit output is the sum of two large voltages and this tends to reduce the effect of an error in either.

Acknowledgment

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