of sufficient amplitude to synchronize an oscillator over the required frequency range, or to provide reconditioned carrier, is transmitted along with the single-sideband signal.

CONCLUSIONS

The use of a common-reference frequency signal in lieu of locally generated oscillations to stabilize radio transmitters and receivers appears to offer a good possibility of obtaining minimum spacing of communications channels in the vhf and higher frequency bands. This technique should provide a large increase in the number of radio channels without changing the essential nature of the transmissions, and at the same time should offer the opportunity for a further increase through the use of single-sideband modulation and other band-reducing techniques. The adoption of this method of communication should result in the release of additional frequency spectra for wide-band applications, such as FM and television broadcasting, in addition to greatly increasing narrow-band facilities of all types in the higher frequency bands.

ACKNOWLEDGMENTS

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Cascade Connection of 90-Degree Phase-Shift Networks*

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***Summary*—A method of connecting 90-degree audio phase-difference networks for use in selective-sideband transmission and reception is shown, whereby an over-all performance is obtained which is analogous to the cascade operation of conventional filters. Three networks are required to obtain twice the rejection in decibels of one. For best results two of the networks must be accurately matched. The cascade connection may either be used with three identical networks to deepen the rejection obtainable with one network over the design range, or it may be used with two similar and one dissimilar network to extend the frequency range over which a given rejection may be obtained.**

INTRODUCTION

NINETY-DEGREE audio phase-shift networks may be used in radio transmission and reception to achieve by modulation methods a selectivity equivalent to that customarily obtained by means of passive filters.[1] Two basic types of these networks have been disclosed. In one, the phase difference between the two output voltages is for all practical purposes 90 degrees, and the relative amplitudes are made as nearly equal as possible;[2,3] in the other, the relative amplitudes are inherently equal, and the desired phase difference is approximated.[4]

In ordinary filter practice, when adequate selectivity cannot be obtained in a single unit, two or more may be connected (with suitable isolation) in series. Thus, if one filter has a 20-decibel rejection in its stopband, a second will provide an over-all rejection of 40 decibels.

While the cascade connection is obvious in the case of conventional filters, it is not readily evident how an equivalent result may be obtained with selective systems using audio phase-shift networks. The problem is relatively easier with the first type of network mentioned above, and one possible approach has been published.[5] It is the purpose of this paper to describe how the other, or difference-phase, type of network may effectively be connected in cascade.

BASIC METHOD

A block diagram is shown in Fig. 1. The material within the upper dotted enclosure represents a single-sideband generator employing audio phase-shift networks. Expressions describing the voltages at each point in the circuit are shown. It will be assumed for simplicity in the following that the radio-frequency phase shift is always exactly 90 degrees and that the audio networks have unity transmission and a phase shift which fails to be 90 degrees by some value $\pm\alpha$,

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[1] I. F. Macdiarmid and D. G. Tucker, "Polyphase modulation as a solution of certain filtration problems in tele-communications," *Proc. IEE* (London), pt 3, vol. 97, pp. 349–358; September, 1950.

[2] R. V. L. Hartley, U. S. Patent No. 1,666,206; April 17, 1928.

[3] B. Lenehan, "A new single-sideband carrier system for power lines," *Elec. Eng.*, vol. 66, pp. 549–592; June, 1947.

[4] R. B. Dome, "Wideband phase shift networks," *Electronics*, vol. 19, pp. 112–115; December, 1946.

[5] H. Chireix, U. S. Patent No. 1,946,274; February 6, 1934.

corresponding to networks of the difference-phase type. To simplify presentation further, all the phase shift is shown lumped in one side of each branch.

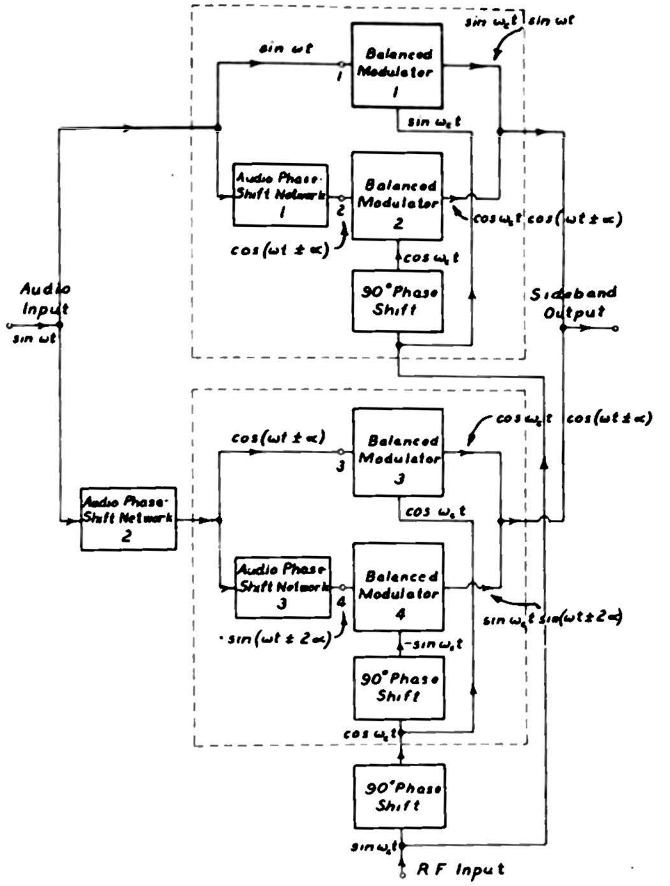

Fig. 1—Connection for obtaining equivalent of cascade operation in single-sideband transmission.

In the upper enclosure of Fig. 1, the output of the first balanced modulator is

$$\sin \omega_c t \sin \omega t = \tfrac{1}{2} \cos (\omega_c - \omega)t - \tfrac{1}{2} \cos (\omega_c + \omega)t, \quad (1)$$

where ω_c is the radio frequency and ω is the audio frequency. That of the second is

$$\cos \omega_c t \cos (\omega t \pm \alpha) = \tfrac{1}{2} \cos \left[(\omega_c - \omega)t \mp \alpha\right] + \tfrac{1}{2} \cos \left[(\omega_c + \omega)t \pm \alpha\right]. \quad (2)$$

When $\alpha = 0$, the upper sidebands cancel, and the lowers add exactly. When $\alpha \neq 0$, a resultant upper sideband R_1 appears whose magnitude and phase is

$$R_1 = \sin \frac{\alpha}{2} \cos \left[(\omega_c + \omega)t \pm \left(90° + \frac{\alpha}{2}\right)\right]. \quad (3)$$

Now the resulting unwanted sideband, caused by imperfect performance of the phase-shift network, may itself be balanced against a similar resultant derived from a second audio network and pair of balanced modulators. The arrangement is shown in the remainder of Fig. 1. The lower dotted enclosure is similar to the upper, but the audio- and radio-frequency voltages fed to it have each received an additional 90-degree phase shift. Let it be assumed that the three audio-frequency networks are identical. The sideband output of balanced modulator number 3 is

$$\cos \omega_c t \cos (\omega t \pm \alpha) = \tfrac{1}{2} \cos \left[(\omega_c - \omega)t \mp \alpha\right] + \tfrac{1}{2} \cos \left[(\omega_c + \omega)t \pm \alpha\right]. \quad (4)$$

That of number 4 is

$$\sin \omega_c t \sin (\omega t \pm 2\alpha) = \tfrac{1}{2} \cos \left[(\omega_c - \omega)t \mp 2\alpha\right] - \tfrac{1}{2} \cos \left[(\omega_c + \omega)t \pm 2\alpha\right]. \quad (5)$$

The resultant of the two upper sidebands is

$$R_2 = \sin \frac{\alpha}{2} \cos \left[(\omega_c + \omega)t \mp (90° - \alpha)\right]. \quad (6)$$

The combination of (3) and (6) yields

$$\text{Undesired sidebands} = R_1 + R_2 = 2 \sin^2 \frac{\alpha}{2} \cos (\omega_c + \omega)t. \quad (7)$$

$$\text{Desired sidebands} \approx 2 \cos (\omega_c - \omega)t.$$

The ratio of magnitudes of undesired to desired sidebands in the output is approximately $\sin^2 \alpha/2$, whereas in one enclosure alone it is $\sin \alpha/2$. Thus, three phase-shift networks, connected as shown, can provide twice the rejection (in decibels) of one network alone.

The actual connection for difference-phase networks is shown in Fig. 2. The branches A_1, B_1, and the like represent the component lattice networks whose constant-magnitude output voltages differ in phase by approximately 90 degrees.

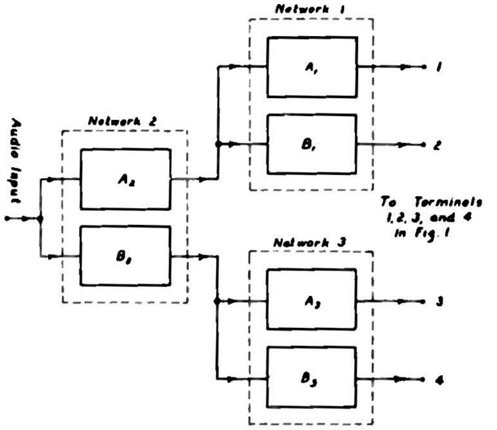

Fig. 2—Connection for networks of the difference-phase type.

It will be found that it is not mandatory to secure the improved undesired-sideband rejection on the output side of the balanced modulators, although this may in some cases be desirable; the improvement may also be obtained directly at audio frequencies. One is led to the circuit of Fig. 3 by analogy to Fig. 1. The two output voltages in Fig. 3 are now found to differ in phase by exactly 90 degrees, but the imperfect phase shift in the networks results in the voltages differing in magnitude by an amount equal to $(1 - \cos \alpha)$. When two such voltages are fed to a balanced modulator, the ratio of desired to undesired sidebands may be shown to be approximately $\sin^2 \alpha/2$. Thus, the circuit of Fig. 3 leads to the same result as Fig. 2, but in a slightly different way.

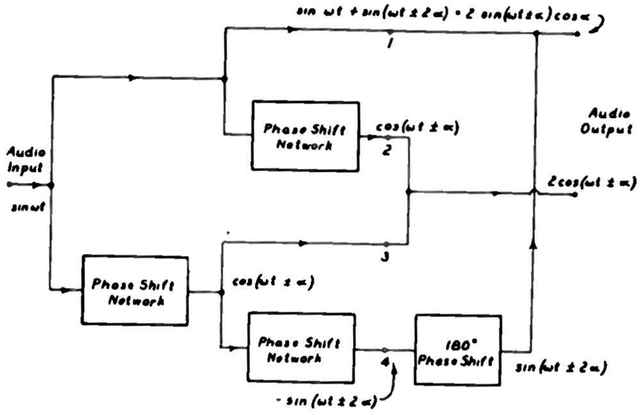

Fig. 3—Audio-frequency equivalent of Fig. 1.

These same circuits are readily adaptable to single-sideband reception.[6,7]

Matched-Performance Requirement

The success of the cascade connection depends on the equality of the phase-shift of networks 1 and 3 in Fig. 1. If these networks are identical, the undesired-sideband output of the first pair of balanced modulators will have the correct phase relationship with respect to that of the second pair to permit best cancellation with the aid of audio network 2. If the shift in network 1 does not match that of network 3, the over-all rejection obtainable will be seriously reduced.

Phase-difference networks may be realized in two ways, either as lattice filters incorporating passive elements only or as half-lattice filters with isolation and phase inversion supplied by vacuum tubes.[4]

With passive networks, the requirements of cascade operation can be met by matching each component of network 3 to the corresponding component in network 1 irrespective of the exact absolute values.

Networks incorporating vacuum tubes may be adjusted for equality of performance to a high degree of accuracy with the aid of connection illustrated in Fig. 4.

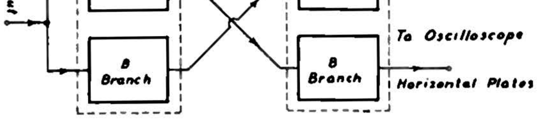

Fig. 4—Connection for making performance of two difference-phase networks virtually identical.

If both have equal phase shifts, the phase difference between the outputs is zero and is displayed as a straight line on the oscilloscope. One network may readily be matched to the other by trimming its time constants until a straight line is obtained at all frequencies of interest. This indication is, of course, much more accurate than the circular 90-degree counterpart.

Extension of Frequency Range

In compensation for the requirement that networks 1 and 3 in Fig. 1 be matched is the circumstance that network 2 may be entirely different. Thus the cascade connection may also be used to give a wider total band over which acceptable performance is obtained, rather than an improvement in performance in a given band. For example, the phase-shift networks of the General Electric type YRS-1 single-sideband adapter afford a rejection of the order of 30 decibels from 70 to 7,000 cycles, a frequency ratio of 100 to 1. Three such networks in cascade should deepen the rejection over this range to something of the order of 60 decibels. However, if network 2 is redesigned to operate from 7,000 to 700,000 cps, the three could then be connected in cascade to give a 30-decibel rejection between 70 and 700,000 cps, a frequency ratio of 10,000 to 1, provided only that the low-frequency networks are able to maintain uniform transmission and similar phase characteristics up to the highest frequency.

Laboratory Test

A laboratory test was performed using the phase-shift and summing stages of three identical YRS-1 adapters connected in the receiving equivalent of Fig. 3. Undesired-sideband rejection was measured by feeding in two test audio voltages having equal amplitudes and a very accurate 90-degree phase difference. The transmission, when all three sideband selector switches were thrown to the passed "sideband" (or phase rotation), was compared with that when the three switches were thrown to the rejected "sideband." The results are shown in Fig. 5.

Networks 1 and 3 were adjusted for equal performance in accordance with the procedure of Fig. 4. Care must be exercised in setting the amplitude balance controls in networks 1 and 3, since an incorrect amplitude setting can sharply affect the phase as well as the amplitude of the partially suppressed "sideband" passed by these units, and thus the over-all rejection.

Conclusion

An arrangement has been shown whereby systems deriving their selectivity from the use of modulation techniques and 90-degree audio phase-difference networks of finite quality may be so connected that an improved over-all performance is obtained in a manner similar to the series connection of conventional filters.

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[6] E. I. Green, U. S. Patent No. 2,020,409; November 12, 1935.

[7] O. G. Villard, Jr., "Simplified single-sideband reception," *Electronics*, vol. 21, pp. 82–85; May, 1948.

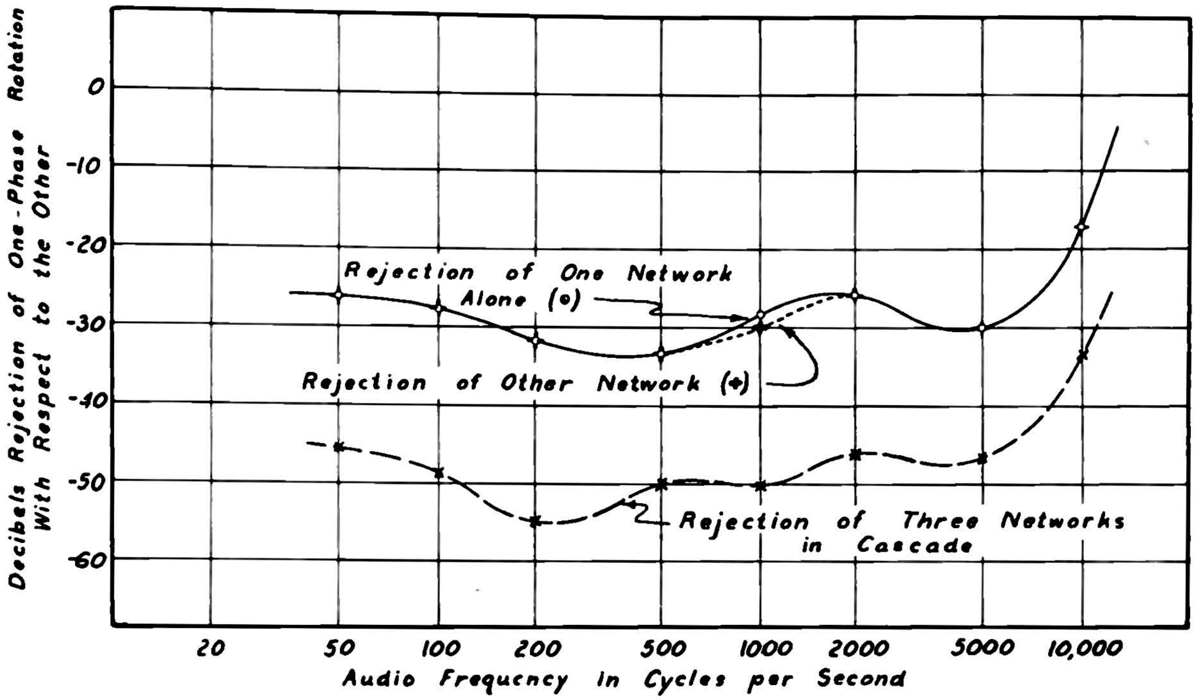

Fig. 5—Rejection obtainable with cascaded commercial 90-degree phase-shift networks. (Individual units part of General Electric Type YRS-1 selector. Nominal frequency range 70 to 7,000 cps.)

Now that the design of RC phase-difference networks for any desired performance is well understood,[8,9] one of the few remaining problems in obtaining high selectivities by this means has been the difficulty of obtaining network components of the desired tolerance. Since resistors and condensers are relatively compact and inexpensive, it may prove desirable in some cases to attain the end result by means of cascaded networks employing relatively lower-tolerance components.

ACKNOWLEDGMENT

The assistance of J. P. Paddock in carrying out the experimental work is gratefully acknowledged.

[8] S. Darlington, "Realization of a constant phase difference," *Bell Sys. Tech. Jour.*, vol. 24, pp. 94–104; January, 1950.

[9] H. J. Orchard, "Synthesis of wideband two-phase networks," *Wireless Eng.*, vol. 27, pp. 72–81; March, 1950.

CORRECTION

Richard Guenther, author of the paper, "Radio Relay Design Data 60 to 600 mc," which appeared on pages 1027–1034 of the September, 1951 issue of the PROCEEDINGS OF THE I.R.E., has brought the following corrections to the attention of the editors:

In Table I, at line "PM or FM," the wide-band gain should be $1/4(B_{rf}/2f_s)^3 = m^3$ instead of $1/4(B_{rf}/2f_s)^2 = m^2$.

The curve for FM in Fig. 7, therefore, runs parallel to the PTM curve according to the corrected figure.

This modifies the results for the two FM examples in Table II (b). According to the increase of the wideband gain from 6 to 12 db, the last line should read: S/N per channel 46 db (instead of 40 db), 36 db (instead of 30 db), 48 db (instead of 42 db), 13 db (instead of 7 db).