The Hall Effect and its Application

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SYNOPSIS:

Semiconductor research and development, now more and more advancing into the broad field of intermetallic compounds, focuses our attention upon a long time known but little explored phenomenon, the Hall-effect.

The construction and application of so-called Hall-generators is discussed herein.

INTRODUCTION:

Semiconductors, in general, belong to the class of materials which are called non-magnetic. This means that no permanent magnetism can be induced and that, in general, a magnetic field feels that there is no difference between a semiconductor and air. A magnetic field is the description in space of the force which would be exerted on a single pole of a magnet by a magnet whose field is being described. An alternative, but entirely equivalent, definition of magnetic field is in terms of the force exerted on a single turn of a coil in which a specified current is flowing.

How do magnetic fields effect semiconductors? There is little to be noticed in the case of only a magnetic field acting on the material, but when an electrical current is passed, there are several interesting effects. This will be apparent when we recall that the force that drives an electric motor is provided in the interaction of a magnetic field and an electric current. If the magnetic field exerts a force on the current in a wire, what is happening in terms of the current carriers in the wire? The answer is that a moving charge carrier will experience force in a magnetic field which will draw it to one side. This can be observed in the action of a magnetic deflection yoke on the beam in a picture tube. We will consider the situation inside a piece of semiconductor in which a current is flowing. The current consists of moving charge carriers which experience a force moving them to the side of the sample. The charges pile up at the surface as in a charged condenser. This surface charge creates an electric field in the bulk of the semiconductor which exerts a force opposite to that of the magnetic field. The consequence of this is that the deflection of charge carriers by the magnetic field will continue until the electric field force created by the charging of the surface is equal to the deflection force of the magnetic field. This constitutes a condition of equilibrium which can be expressed as follows:

$$\text{Magnet deflection force} = qv_xH_z = \frac{J_x}{n}H_z$$

$$\text{Electric field force} = q^{E}H = q\frac{V_H}{w}$$

n = number of charge carriers to unit volume
q = electrical charge of each carrier
v_x = velocity of charge carriers along the sample
H_z = magnetic field strength perpendicular to the sample
J_x = current density along the sample

E_H = electric field produced by the charge built up on the surface

w = width of a rectangular sample

t = thickness of a rectangular sample

V_H = voltage due to the charge build up, called the Hall voltage

(Hall-voltage after E. H. Hall who discovered it in 1879)

Setting the two force equal as the condition for equilibrium:

$$\frac{q}{w} V_H = \frac{J_x H_z}{n}$$

$$V_H = \frac{1}{nq} J_x W H_z$$

This can be expressed in terms of the total current by considering that $I_{TOT} = -J_x A$.

$$V_H = \frac{1}{nq} \frac{I_x H_z}{t} = R \frac{I_x H_z}{t}$$

R is the Hall constant and characterizes the density of charge carriers in the material.

$$R = \frac{1}{nq}$$

This measurement is made on a material to determine the charge carrier concentration, which is equal to the impurity concentration. The further value of this measurement can be understood by considering that this number n and the conductivity σ will allow us to solve for the mobility

$$R \sigma = \frac{1}{nq} \cdot nqu = \mu$$

Thus the measurement of conductivity and Hall constant will give us the two constants of the material which characterize the electric conduction in fundamental terms.

Another effect which occurs in a magnetic field is the magneto-resistance. If the resistance of a sample is measured in a magnetic field and again with no magnetic field, it will be found that the resistance is higher in a magnetic field. And experimentally, it has been determined that

$$\frac{R - R_o}{R_o} = \mu^2 H^2$$

R_o = resistance with no magnetic field

R = resistance with an applied field

μ = mobility

H = magnetic field

This change of resistance is due to a change of the mobility of the charge carriers. The charge carriers will feel an additional force as they traverse the path between collisions. This force (the same which sweeps carriers to the sides of the sample in the Hall effect) will cause the carriers to move in a curved path between collisions; the net effect of this is to reduce the velocity in the direction of current flow. In this case, as in the Hall effect, the magnetic field is to be perpendicular to the current flow to cause the effect.

Changes of up to 100:1 in resistance may be induced by magnetic fields of 30,000 gauss.

A large number of applications of the Hall effect and magneto-resistance are possible by the use of compound semiconductors such as In Sb and In As in which the mobility of the electrons is very high.

MATERIAL PREPARATION:

It has been known for quite some time that various A^{III} -B^{V} compounds exhibit semiconductor properties. The element A^{III} represents a material of the third main group in the periodic table, for example In (Indium) and the B^{V} stands for a material of the fifth main group Sb (Antimony) for example. It should be stressed, however, that A^{III} -B^{V} combinations are considered intermetallic compounds, and the semiconductor characteristic is not an inherent property of all of these materials. There is a rather large number of compounds considered pure metals.

Since the objective of this paper is the discussion of the Hall-generator, we are mainly concerned with two compounds.

I_n S_b Indiumantimonide

and

I_n A_s Indiumarsenide

The preparation of these materials is similar and can be accomplished by melting together both components. The indium and antimony, for example, is obtained by the reduction of their oxides with potassium cyanide. The reduced oxides are first purified by chemical means, so that the prepared indium contains about 0.01% of impurities (several, like Ca, Mg, Ag, Al and Sd can be determined), and the antimony shows only traces of Cu in quantities less than 0.001%.

The compound produced from both components is then subjected to zone-recrystallization. Monocrystals are drawn from zone-melted material and cut with diamond blades in thin wafers of a thickness ranging from 10 - 20 mils.

The semiconductor properties of the material are characterized by determining the dependence of the conductivity and the Hall-constant on the temperature, and by determining the type of conductivity by the sign of the thermal emf at the temperature of liquid air.

DEVICE FABRICATION:

The wafers of the $A^{IV}-B^{V}$ compound are generally 10 to 20 mils thick. As we will see later, the thickness of the Hall-generator is a vital factor. The thinner the Hall-generator can be made, the smaller the air gap can be and, consequently, less current is required to produce a magnetic field of a determined strength.

The intermetallic compound is a very brittle material. To increase the mechanical strength, the $I_n S_b$ or $I_n A_s$ wafers are mounted on so-called back-plates. The selection of the back-plate material is somewhat difficult because the temperature expansion coefficient of the material has to be the same as that of the compound, and yet the material has to provide the necessary mechanical strength. Before the wafers are glued on the back-plate, they are lapped down to a desired thickness. This is a delicate procedure and requires quite some skill.

After the lapping, the wafers are etched in CP4 and dried. The so-prepared wafers are glued onto the back-plate and, after that, cut with a diamond wheel or sandblasted to obtain dices of the desired size.

The Hall-generator is normally of a size corresponding to a side-relation of 2:5.

The ohmic contacts are then soldered on with tin and resin.

It has to be observed that the leads or contacts for the control current are applied over the full length of the small sides of the device. The contacts for the Hall-voltage have to be connected exactly on the center line and on the opposing, long sides of the slab. Since this is very difficult to achieve, two contacts are provided on one side to enable centering with a variable resistance.

The leads, where the Hall-voltage is taken off, have to be twisted in order to cancel any additional voltage possibly induced by the magnetic field.

The Hall-generator prepared in the described fashion can be sprayed with a laquer, or incased in epoxy resin, which of course leads to more problems, but has its merits.

APPLICATIONS:

A Hall-generator is essentially a device that provides a voltage output proportional to the product of two quantities - the current flowing through the device, and the magnetic field penetrating the device perpendicularly.

A properly designed and built Hall-generator can have an output exactly proportional to the product of the magnetic field strength and the current. This capability to calculate a product suggests uses as an analog computer element. This consideration can be taken in account for frequencies up to 10^{14} cps.

Similarly, an electrical quantity can be squared by such an analog element. The quantity to be squared is simply expressed both as a field and as a current. Under these conditions, application of the electrical quantity to the input of the device will yield a Hall-voltage proportional to the square of the input parameter.

If the magnetic field penetrates the Hall-generator at some angle other than 90 degrees, the output of the Hall-generator is proportional to the magnetic field times the cosine of the angle between the normal to the Hall-generator plane and the magnetic flux lines. This method gives a very precise and simple method for obtaining an electrical analog of the cosine or sine of a mechanical rotary displacement. This analog automatically goes through zero and produces polarity reversals in different quadrants.

This type of Hall-generator can also be used as a position indicator. A magnetic field is set up and a constant control current is sent through the generator. The output of the generator will be some function of the distance of the generator from the magnetic pole. In this way, the generator can act as an indicator of position.

If a Hall-generator is placed in the air gap of a split C-yoke, it is possible, by placing the yoke around the bus-bar, to measure high bus-bar currents without breaking the current path and without many of the inherent difficulties with the presently used equipment.

Because of their extremely fast response time, Hall-generators can be used to measure the power content of transients. In plots of fuse blow-out or lightening-arrestor breakdown, the current and voltage surges do not coincide. To calculate the maximum power developed in the device during a fault, the appropriate voltage and current curves are multiplied, point by point. The fault voltage or a proportional fraction thereof is impressed on a Hall-generator and the fault current generates a magnetic field. The Hall-voltage then produces a trace on an oscillograph which is the equivalent to the product of these two quantities and thus is proportional to the power content of the fault pulse. By similar circuitry, an economical watt-meter, without moving parts, can be devised.

Hall-generators can be used to measure the internal torque of d-c motors. This is most simply accomplished by placing the device in the air gap on the surface of one of the poleshoes. A current, proportional to the armature current, is fed through the Hall-generator. Since the internal torque of a d-c machine is proportional to the product of the armature current and the flux density in the air gap, the Hall-generator produces a voltage proportional to the internal torque.

APPLICATION EXAMPLES:

SAW-TOOTH GENERATOR

Description: Saw-tooth generators in use at present have certain undesirable characteristics, for example, the instability.

The discharge tube (controlled or not) is an electronic device and, therefore, subject to changes in its characteristic. Since the tube is part of the tuned circuit, the consequences are obvious.

Another important factor is the retrace, or fly-back time. In T.V. application, as short as possible retrace time is desirable. This cannot be achieved with an electronic tube. The internal impedance of a tube is much too big to allow a fast discharge of the capacitor in the circuit.

The circuit described in this disclosure overcomes these limitations.

Figure 1 shows a Hall-generator placed in a magnetic field which is supplied by a square wave generator. The magnetic flux B will be proportional to the current i_B in the coil. The control current i_C for the Hall-generator is taken from the same square wave supply. R is a current limiting resistor.

Figure 2 demonstrates the wave shape of the Hall-voltage which is proportional to the product of B and i_C. During the positive part of the square wave cycle, the current in the coil increases linearly. The current in the Hall-generator has a constant positive value. As a result, the Hall-voltage increases linearly from $-V_{H1}$ to $+V_{H1}$.

When the square wave changes from positive to negative, the current in the Hall-generator, but not the current in the coil, are able to follow. In consequence, the polarity of the Hall-voltage V_{H1} is reversed from $+V_{H1}$ to $-V_{H1}$. Thus the saw-tooth wave is seen to have an almost ideal, that is a vertical fly-back.

During the negative part of the square wave cycle, the current in the coil decreases linearly. The current in the Hall-generator has a constant negative value. As a result, the Hall-voltage again increases linearly from $-V_{H1}$ to $+V_{H1}$. Then the Hall-voltage is quickly reversed to $-V_{H1}$ and the cycle starts again.

Advantages: 1. This circuit is not tuned and the frequency stability is the same as that of the square wave generator.

2. The wave shape of the saw-tooth voltage is close to ideal because of an almost vertical fly-back.

3. Temperature and humidity of the ambient has little effect on the conversion of the square wave input to a saw-tooth output.

4. The output of this converter has no d-c component - another desirable factor when the output is fed to a transformer.

Note: Disclosed by W. Prenosil to Westinghouse El. Corp, Pittsburgh, Pa.

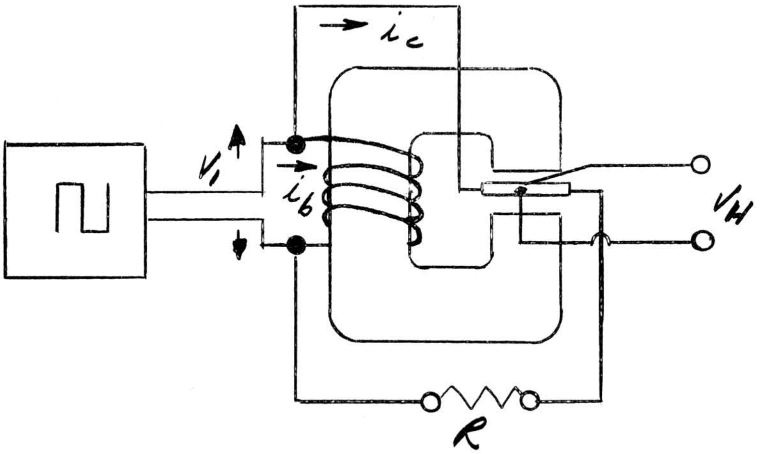

Fig. 1

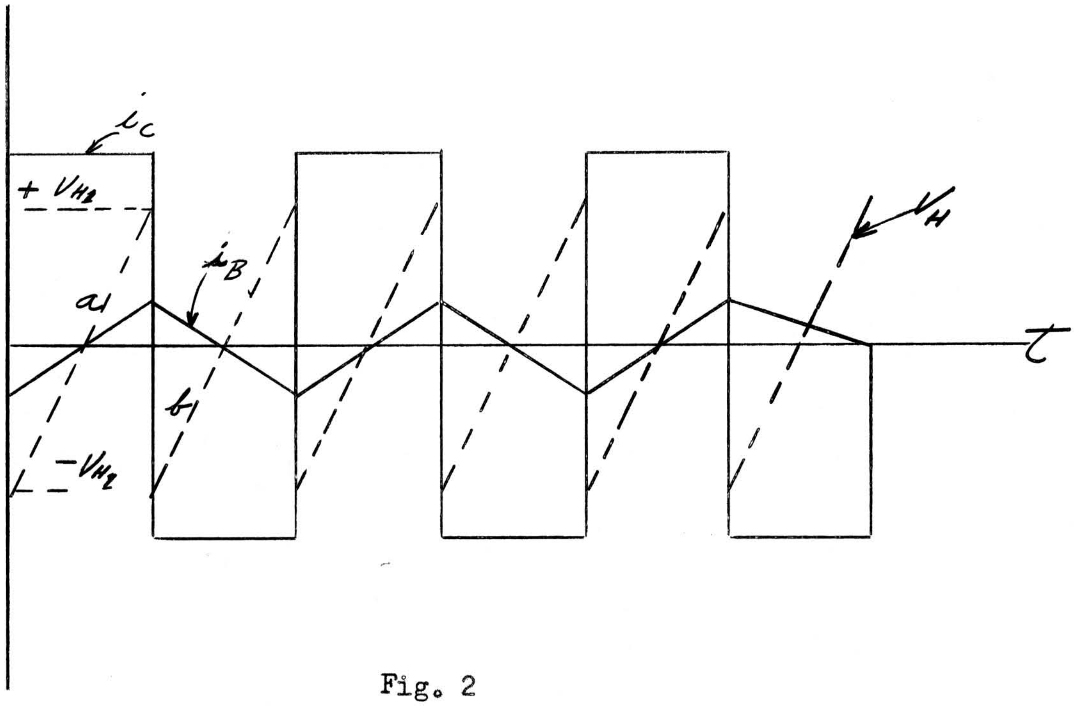

Fig. 2

FREQUENCY MIXING AND MULTIPLICATION:

Description: Frequency converters or multipliers in use at present have the disadvantage of employing tuned output circuits. Changes in the load or the value of components may change the wave form and efficiency considerably. The circuit described in this disclosure overcomes this disadvantage.

A Hall-generator is placed in an electromagnetic field and is supplied with a control current. Figure 3 shows the arrangement of the circuit. The current i_f flows through the coil C and creates the magnetic field B. The control current i_c for the Hall-generator may either be supplied by the same signal source as the field, or by another signal source. Two distinctly different results are obtained in these two cases.

The output voltage of a Hall-generator is given by the expression

$$V_H = K \cdot B \cdot i_c$$

Let $i_f = i_{fo} \cdot \sin(\omega_1 \cdot t + \varphi_1)$ (1)

Then $B = B_o \sin(\omega_1 \cdot t + \varphi_1)$ (2)

And let $i_c = i_{co} \cdot \sin(\omega_2 \cdot t + \varphi_2)$ (3)

Then the Hall-voltage becomes

$$V_H = K \cdot i_c \cdot B_o \cdot \sin(\omega_1 \cdot t + \varphi_1) \sin(\omega_2 \cdot t + \varphi_2) \quad (4)$$

$$= \frac{1}{2} K \circ i_c \circ B \left\{ \cos\left[(\omega_1 - \omega_2) \quad t + (\varphi_1 - \varphi_2) \right] - \cos\left[(\omega_1 + \omega_2) \quad t + (\varphi_1 + \varphi_2) \right] \right\} \qquad (5)$$

The Hall-voltage is seen to contain the sum and difference of the two signal frequencies.

1. Frequency Mixing

In the case of frequency mixing, the signals are supplied by different sources and we have

$$\omega_1 \neq \omega_2$$

Thus expression (5) leads to the result of the two frequencies $(\omega_1 - \omega_2)$ and $(\omega_1 + \omega_2)$

An adequate filter may be used to select either one of the two frequencies - $(\omega_1 - \omega_2)$ and $(\omega_1 + \omega_2)$.

2. Frequency multiplication

If the currents i_f and i_c are supplied by the same source,

$$\omega_1 = \omega_2 = \omega$$

The Hall-voltage is then

$$V_H = \frac{1}{2} K \cdot i_{co} \cdot B \quad \cos [(\varphi_1 - \varphi_2) - \cos (2\omega t + \varphi_1 + \varphi_2)] \quad (6)$$

If furthermore the phase difference between the two signals is adjusted by means of a phase shift network Ph (see Figure 4), so that

$$\varphi 1 - \varphi 2 = \frac{n\pi}{2} \quad \text{where } n = 1; 3; 5 \ldots \quad (7)$$

then the Hall-voltage will be

$$V_H = K \cdot i_{co} \cdot B \cdot \frac{1}{2} \cos (2\omega_1 t + \varphi_1 + \varphi_2) \quad (8)$$

Advantages: 1. This circuit does not employ a tuned output circuit. In consequence, change in load has little effect on the wave form.

2. By means of a tunable phase shift network, a wide range of frequencies can be covered.

3. Frequency mixing can be obtained over a very extended range without tuning.

4. This means of frequency mixing or doubling does not create d-c components which is of importance when the output is fed into a transformer with an iron core.

5. Because of the linear characteristic of the Hall-generator, the harmonic distortion will be negligible.

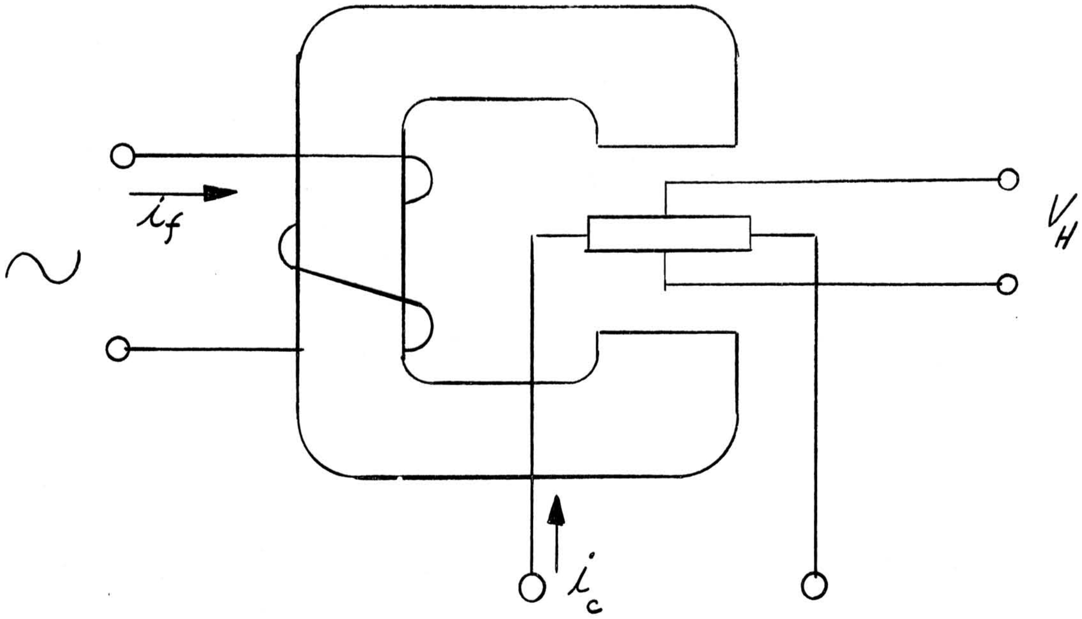

Fig. 3

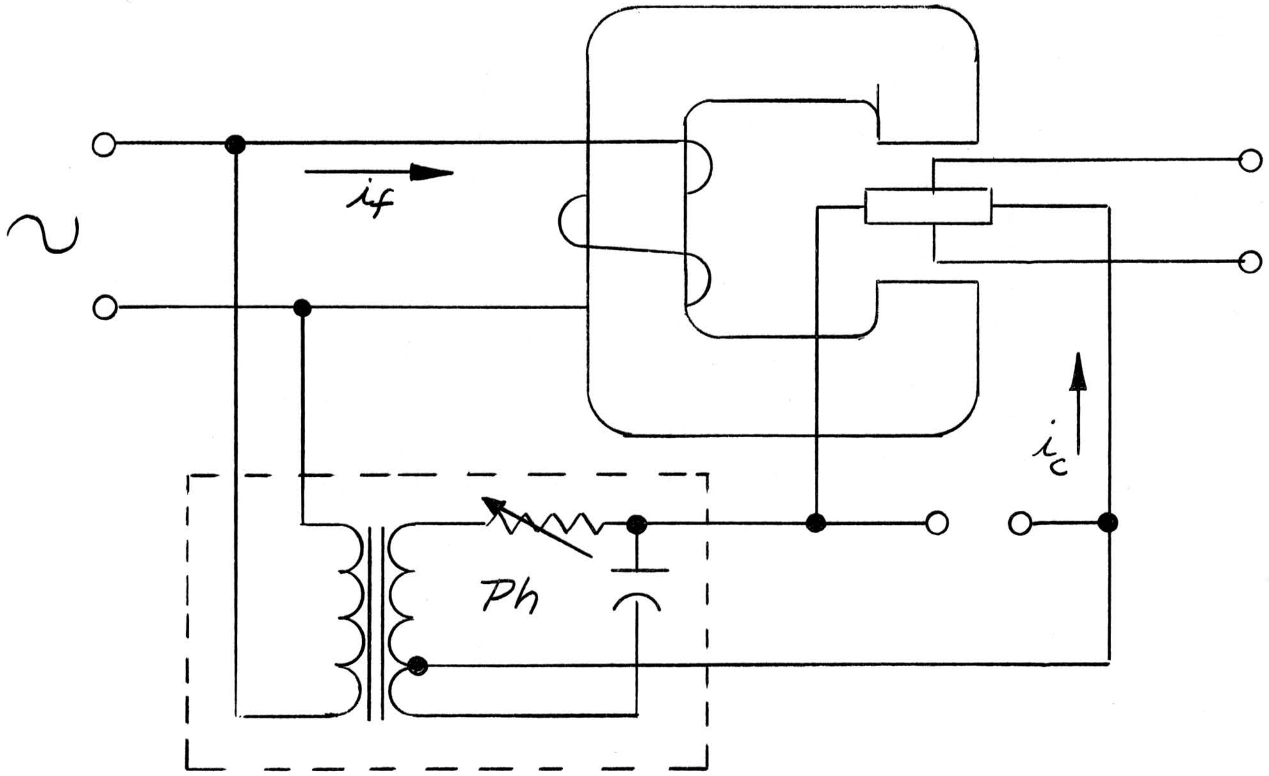

Fig. 4

POLY-PHASE GENERATOR OR MODULATOR:

Description: For many tests and investigations, poly-phase generators and modulators are needed. The power level at which such devices work is relatively low, but it is important for the generator to have an undistorted sine wave of stable frequency with a given and unchangeable angle between phases. This condition can be achieved to a certain extent with a conventional generator. Unfortunately, such a generator has several disadvantages. These are (1) frequency dependency of output level, (2) the various components employed react differently to temperature changes, and (3) recalibration is necessary after just a minor change in working conditions or components.

The device described herein is useful in converting d-c to a-c with a theoretically unlimited number of phases, provided only that separate d-c supplies and, at higher frequencies, adequate chokes in the control current circuit are used. Figure 5 and 6.

The device also works as a modulator.

A number of Hall-generators are mounted on the shaft of the motor, with variable and controlled speed, corresponding to the number of phases and in a position corresponding to the phase angle desired. Also provided on the shaft are the necessary slip rings to supply the control current to the Hall units and to take off the Hall-voltage.

The Hall-probes mounted on the motor shaft rotate in a magnetic field whose strength is variable permitting variation in magnitude of output voltage.

Another possibility is the mounting of the Hall-generator on the armature of the d-c motor. In this case, a separate magnetic field is not necessary.

Still another possibility is to use the setup as a modulator by varying the speed of the motor or the field strength.

Amplitude modulation is achieved by varying the field strength or the control current.

Frequency modulation is obtained by varying the speed of the motor. Using an a-c field or an a-c control current, the device can be used as a mixer.

Advantages: 1. Hall-generators made from the same material are known not to be equal, but are identical in behavior and have linear characteristics. The only factor to be taken into account as a variable is the temperature dependency which is, as pointed out above, equal in all Hall-generators of the same size and material. In consequence, a correction factor for all units can easily be introduced.

2. Frequencies from 0 up to 100 kc may be generated, and the frequency range is only limited by the construction of the motor or the gears employed.

3. Frequency can be changed without altering magnitude of output voltage.

4. The device is superior to an a-c motor generator, especially at higher frequencies, due to the absence of iron losses.

5. This device is also superior to a static poly-phase generator, because it does not employ tuned circuits and, therefore, is not frequency dependent.

6. The output voltage can be modulated by means of a magnetic field and/or a control current.

7. The frequency can be modulated by varying the speed of the motor.

8. By applying an a-c control current to the Hall-generators and varying the frequency of the a-c voltage applied to the field, frequency conversion can be achieved.

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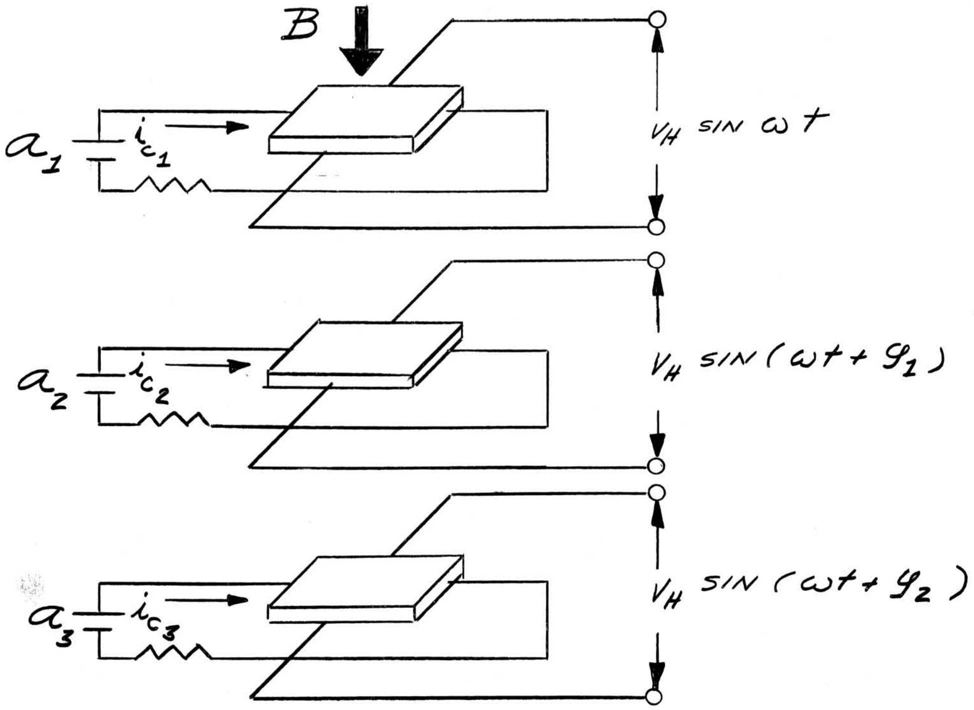
B
a_1
i_{c_1}
$V_H \sin \omega t$
a_2
i_{c_2}
$V_H \sin(\omega t + \varphi_1)$
a_3
i_{c_3}
$V_H \sin(\omega t + \varphi_2)$

Fig. 5

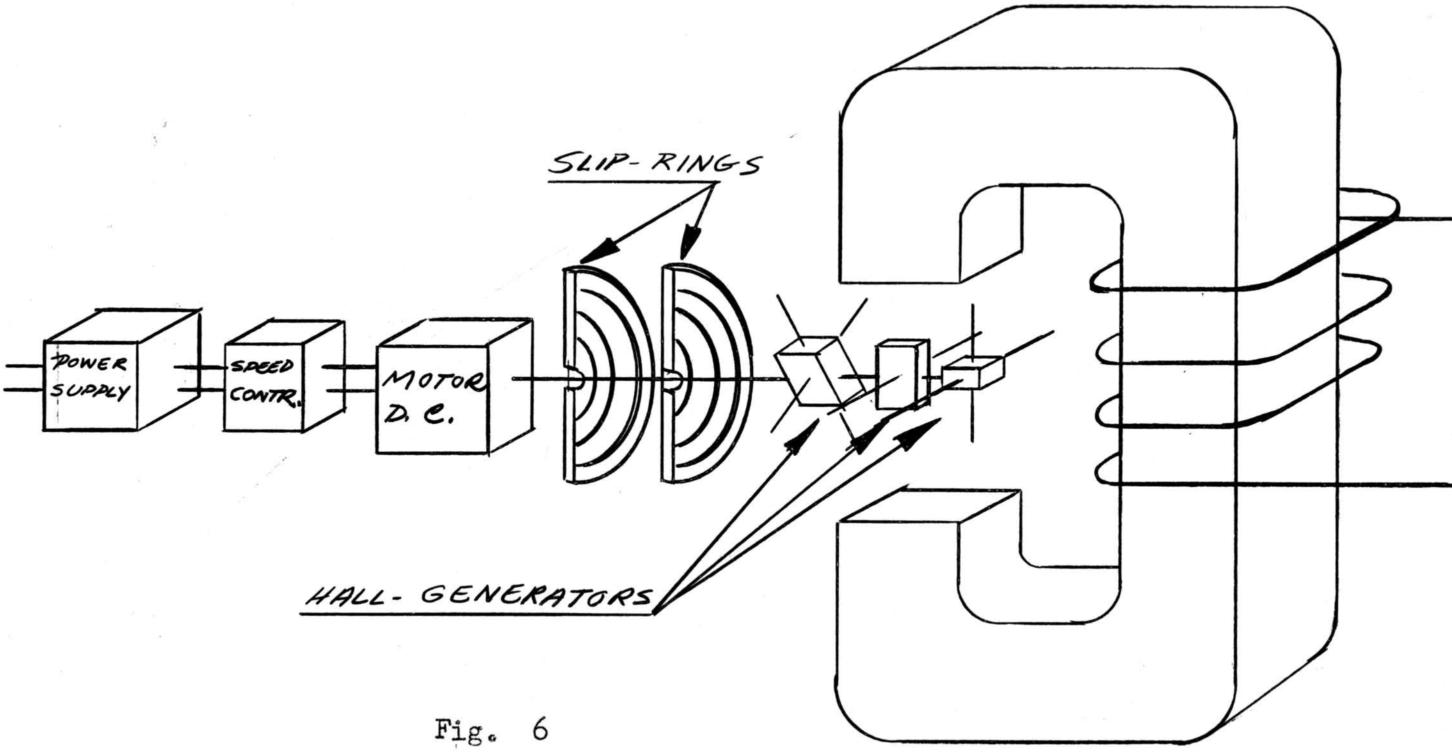
SLIP-RINGS
POWER SUPPLY
SPEED CONTR.
MOTOR D.C.
HALL-GENERATORS

Fig. 6