Conditions in the Anode Screen Space of Thermionic Valves*

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In a recent paper[1] Harries has indicated the difficulties of obtaining useful results from a theoretical consideration of the conditions in the screen-anode space of a thermionic valve. The following elementary treatment of potential distribution in a special case of the planar diode seems to provide an approximation to some practical cases and to agree qualitatively with the experimental results.

Consider the unidirectional streaming of electrons normal to two parallel planes, which are maintained at steady potentials V_1 and V_2 both greater than zero. The electrons leaving the first plane have a uniform velocity v_1 where $\frac{1}{2} mv_1^2 = V_1 e$, and e is the magnitude of the negative electronic charge. We require to investigate the possible types of potential-distance relations. The fundamental equations are:

$$\frac{d^2V}{dx^2} = 4\pi\rho \quad \ldots \quad (1)$$

$$i = \rho v \quad \ldots \quad (2)$$

$$\tfrac{1}{2}mv^2 = Ve \quad \ldots \quad (3)$$

Here ρ and i are the magnitudes of the negative (electronic) space charge density and the electron current density respectively.

Eliminating ρ and v we have:

$$\frac{d^2V}{dx^2} = 4\pi i\sqrt{\frac{m}{2eV}} \quad \ldots \quad (4)$$

and a first integration gives

$$\left(\frac{dV}{dx}\right)^2 = \frac{16K^2}{9}(V^{\frac{1}{2}} - S)i \quad \ldots \quad (5)$$

where S is a constant of integration and we have written

$$K^2 = 9\pi\sqrt{\frac{m}{2e}}$$

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[1] Harries. *The Wireless Engineer* (April, 1936).

Whence $\pm \dfrac{dV}{\sqrt{V^{\frac{1}{2}} - S}} = \dfrac{4K\sqrt{i}}{3} . dx$

A further integration now yields

$$\pm \sqrt{V^{\frac{1}{2}} - S}(V^{\frac{1}{2}} + 2S) = Kx\sqrt{i} + C \quad \ldots \quad (6)$$

where C is another constant of integration. Suppose we take the first plane to be $x = 0$ and the second $x = d$. Then the constants are to be determined from the boundary conditions

$$\pm \sqrt{V_1^{\frac{1}{2}} - S}(V_1^{\frac{1}{2}} + 2S) = C \quad \ldots \quad (7a)$$

$$\pm \sqrt{V_2^{\frac{1}{2}} - S}(V_2^{\frac{1}{2}} + 2S) = Kd\sqrt{i} + C \quad \ldots \quad (7b)$$

Equation (7a) determines C except for an ambiguity of sign. The value of S corresponding to given values of V_1, V_2, d and i can be found graphically. We plot

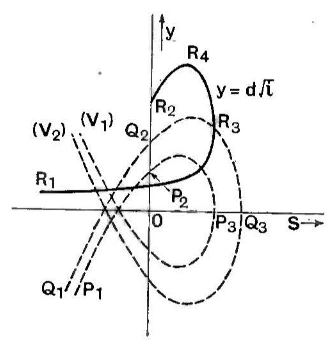

Fig. 1.

$y = \pm \sqrt{V^{\frac{1}{2}} - S}(V^{\frac{1}{2}} + 2S)$ as a function of S for the particular values $V = V_1$ and $V = V_2$. This gives two curves of the type shown dotted in Fig. 1. The quantities d and i occur only in the product $d\sqrt{i}$ whose corresponding values are then obtained by adding or subtracting the ordinates according to the possible combinations of sign indicated in equations (7a) and (7b).

Trial will show that the only acceptable solutions are obtained by adding over the portions P_2P_3, Q_2R_3, and subtracting over the portions $P_3P_2P_1$, $R_3Q_2Q_1$. Drawing the corresponding curves, shown full in Fig. 1, we can immediately read off the appropriate value of S for any value of $d\sqrt{i}$. The potential-distance relation is then obtained by substituting this value of S in equation (6) together with the value of C given by (7a) and choosing appropriate combinations of signs.

Let us consider the general form of this relation. Suppose V_1 and V_2 to be fixed, then three régimes can be distinguished. The first includes the region R_1R_3 of Fig. 1, i.e. all $d\sqrt{i}$ less than a certain value. The potential distance curve is here of a simple monotonic type and calls for no comment. The second comprises larger values of $d\sqrt{i}$ (region $R_3R_4R_2$) and the potential distribution now exhibits a minimum. The parameter S has here a special significance. Referring to equation (5) we see that

$$S = V^{\frac{1}{2}} - \frac{9}{16K^2 i}\left(\frac{dV}{dx}\right)^2$$

At the minimum $\frac{dV}{dx} = 0$ and therefore if the minimum potential is V_m, $S = \sqrt{V_m}$. We note that if we increase the value of $d\sqrt{i}$, the value of S and therefore of V_m decreases over the portion R_3R_4 of the curve. At a certain stage, however, two possible values of V_m are found for each value of $d\sqrt{i}$, corresponding to points on the portions R_3R_4 and R_2R_4 of the curve. Müller[2] has treated a similar case from the point of view of transit time, and concluded that in those cases in which his equations indicated two possible space charge distributions, the one giving a longer electron transit time was unstable. General energy considerations would also suggest that the conditions corresponding to the portion R_2R_4 of the curve cannot be stably realised. For values of $d\sqrt{i}$ greater than that at R_4, there is no solution for our ideal problem. Evidently our simple treatment does not take account of sufficient factors to deal with this third régime.

The effect of varying V_1 and V_2 while the quantity of $d\sqrt{i}$ is kept constant can be traced in a similar manner from a series of curves such as that given in Fig. 1.

The above analysis might be expected to apply as a first and rather crude approximation to the actual conditions in an anode screen space which is receiving a substantially "saturated" current from the cathode space. It takes no account of the inhomogeneity of the field, and the associated electron-optical aspects of the problem.[3] Neither does it consider the spread of electron energies or the presence of secondary emission from either of the electrodes. It seems plausible to guess that if the secondary emission is not too large, its effect will be to enhance the space charge density and alter the ranges but not the main features of the possible space charge distributions deduced for the ideal case. We proceed, then, on the assumption that the departure of the practical from the ideal case is not so great that it invalidates the general nature of the results. For small screen-anode spacings (d small) we should expect, for the operating range of electrode potentials, a potential curve having either no minimum, or a minimum insufficiently pronounced to prevent the passage of secondary emission. The "dynatron" fold would therefore appear in the characteristic. For larger spacings the decrease in V_m would provide a potential barrier to the secondaries, smoothing out the fold. For very large spacings some other condition would ensue, which, presumably, is represented by the characteristic for a 6 cm. spacing shown in Fig. 1 of Harries' paper. Here the anode current is very small for anode potentials less than 200 v. but thereafter rises sharply to its normal saturated value.

[2] Müller. *Hochfrequenztechnik und Elektroakustik* 41 (May, 1933).

[3] For a recent discussion see Rothe and Kleen. *Telefunken-Röhre* 6 (March, 1936).