Mode cutoff conditions and data on transverse decay are given in Table I. The decay data were calculated with the parameters such that the second mode (of its type) was just at cutoff; this gives the most favorable condition (weakest external field) with only the lowest mode propagating. For hr large, $H_1^{(1)}(ihr)$ behaves like $(r/a)^{-1/2}\exp[-(ha)r/a]$, where a is the radius of the rod. This asymptotically exponential attenuation has been expressed in db per radius in Table I.

The H_{11}-like mode in the rod and the H_1 mode of the dielectric plate have zero cutoff wave number. Because of this characteristic they have been described in Table I as principal modes, but they have longitudinal field components and their phase velocities depend upon frequency.

Engineering Applications

Although the external modal field is not in itself radiated, it will induce currents in objects near the guide surface, and hence indirectly cause both reflections and radiation. By reciprocity, power could be received from another system.

If the guide were designed to support a few or several modes, their resultant field could be concentrated near the center. For a given dielectric material and a given frequency this would require a guide mechanically larger than for a single mode, and over any appreciable distance dispersion would still push the field to the surface. In the present state of the art it is difficult to couple in and out of any kind of guide which propagates more than one mode, but we may learn how in the future.

For comparison with the visible light analog, where we know the external field to be very weak, a glass rod 1 cm in diameter, with an index of 1.4 and yellow light, will support 17,000 H_{on} modes.

The dielectric slab might be made useful by bounding it with metal plates parallel to the $x-z$ plane and extending them so far in the x direction that the field at their edges is weak. This is just an involved way of describing a parallel-strip transmission line with a bar of dielectric material to concentrate the field near the center.

The dielectric guide will be useful for transmitting power over short distances where mechanical flexibility and low cost are important, and imperfect shielding can be tolerated.

Acknowledgment

The author is indebted to C. W. Horton of the University of Texas for permission to use his data on the H_{1n}-like and the E_{on} modes to obtain their cutoff values and their transverse attenuations.

The Relationship Between the Emission Constant and the Apparent Work Function for Various Oxide-Coated Cathodes*

HAROLD JACOBS†, GEORGE HEES†, AND WALTER P. CROSSLEY†

Summary—**Measurements were made of the emission of oxide-coated cathodes on six chemically different metal wires over a period of 500 hours. An empirical emission equation was employed which was found to be as accurate as the conventional Dushman equation and to be simpler to use. It was found that the logarithm of the emission constant A' varied as the sum of the apparent work function times a constant, and the constant B.**

Introduction

IN THE HISTORY of oxide-coated cathodes, there has been considerable discussion on the meaning of the constants of the various emission equations. A considerable difference in opinion exists at the present time as to how these constants vary. According to Blewett,[1] Espe[2] concludes that "activation is due to an increase in the A factor while the work function remains constant." Heinze and Wagener[3] claim that activation follows from a decrease in the work function alone. Others, Detels[4] and Huxford,[5] show that the work function and A factor both decrease in such a way that log A is a more or less linear function of the work function. DeBoer[6] theorizes (using a concept of thermal ionization of surface atoms) that the A factor should change and the work function remain constant. Actually, some of the difficulty in attaining unified results and opinions

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† Advanced Development Laboratory, Sylvania Electric Products Inc., Kew Gardens, L. I., N. Y.

[1] J. P. Blewett, "Properties of oxide coated cathodes," *Jour. Appl. Phys.*, vol. 10, pp. 831–848; December, 1939.

[2] W. Espe, "Thermionic constants of oxide coated cathodes," *Zeit. fur. tech. Phys.*, vol. 10, p. 489; 1929.

[3] W. Heinze and S. Wagener, "Variation of emission constant of oxide cathodes during activation," *Zeit. fur. Phys.*, vol. 110, p. 154; 1938.

[4] F. Detels, "Formation processes in oxide cathodes," *Zeit. fur. Hochfrequenz*, vol. 30, pp. 10–52; 1927.

[5] W. S. Huxford, "Photoelectric emission from oxide coated cathodes," *Phys. Rev.*, vol. 38, p. 379; 1931.

[6] J. H. DeBoer, "Electron Emission and Absorption Phenomena," Cambridge University Press, 1935, p. 34.

may reside in studying the constants under different experimental conditions, where, due to different pressures, temperatures, and general conditions, the actual experiments may have differed.

In the work to be reported an attempt was made to observe the changes of the constants of an empirical emission equation

$$I = A' \exp\left(\frac{-e\chi}{kT}\right) \quad (1)$$

over a 500-hour period of life, after the oxide-coated cathodes had been broken down and aged rather completely. Using this equation, we have set up the problem of describing the electron emission from oxide-coated cathodes as a function of life, and to determine whether or not any relationship exists which may be indicated experimentally between the A' factor and the work function. It was decided to use this equation for two reasons: firstly, it is in the simplest form for practical use; secondly, this equation can perhaps have future possibilities, since it can be shown to have some relation to the modern theory of solids.[7]

Description of Experimental Tubes and Tube Processing

To study thermionic electron emission, special emission-test diodes were constructed as sketched in Fig. 1. These tubes contained three grade-A nickel cylinders to be used as anodes in a standard soft-glass "lock-in"-type

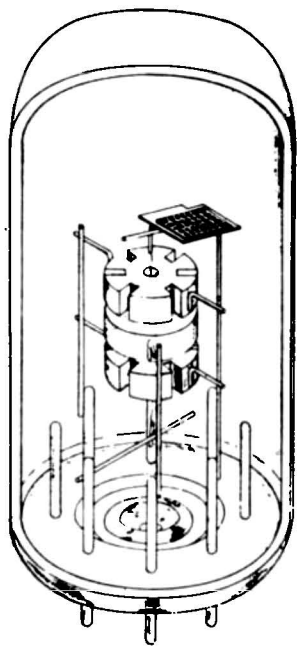

Fig. 1—Experimental diode.

vacuum-tube bulb. The purpose of the slotted end cylinders was to act as guard rings and help provide a uniform electrostatic field at that portion of the filament being studied. The use of guard rings also minimized "end effect"—cooling of the ends of the filament by thermal conduction. The center anode was isolated electrically and used as the collector for the emission studies. To reduce the probability of evolution of gas, no mica or lava spacers are used as is customary in commercial tubes, but all spacing was accomplished by means of nickel supports. The outside diameter of the anode cylinders was 0.500 inch; the inside diameter, 0.125 inch. The center anode was 0.375 inch long, and the guard rings 0.125 inch long. A 0.050-inch peep hole was provided in the center anode so that the brightness temperature of that portion of the filament being studied could be measured directly with an optical pyrometer. A spectral-emissivity factor of 0.6[8] was used as a correction for the emissivity of the oxide coating, and resulted in a correction of 33°C at a color temperature of 900°C brightness.

The filaments consisted of 0.005-inch diameter wire, coated with a standard triple-carbonate suspension to an approximate outside diameter of 0.0065 inch. The coating density was held within 1.0 ± 0.3 grams per cubic centimeter.

The diodes are exhausted, activated, aged, and life-tested under identical conditions to eliminate, to the greatest extent possible, the effects of these factors on the ultimate thermionic emission. Table I gives exhaust, activation, and aging schedules used, and Table II gives the life-test conditions.

Methods of Measurement

Data to plot the emission curves were obtained in the filament-temperature ranges of 850 to 1100°K by raising the plate voltage, and recording plate current and voltage until temperature-limited current was drawn. The highest filament temperature was, in general, that which gave a temperature-limited current of 12.0 ± 2 ma at a plate voltage of 200.0 volts dc. It has been observed that the filaments can be heated markedly by the cathode current being drawn, and because of this it was sometimes necessary to run emission curves at initial currents less than 12.0 ma.

The filament temperature in the region below 1050°K was determined from the filament volt-amperes by extrapolation on a temperature calibration curve. Filament temperature was measured as a function of filament volt-amperes at several points in the visible temperature region (900, 860, 820, and 790°C brightness) and extrapolated to room temperature at 0 volt-amperes input.

The logarithm of the plate current was plotted against the plate voltage. The slopes of the space-charge region and the temperature-saturation region were extrapolated. The point of intersection of these projected lines was taken as the point of zero effective field on electrons leaving the filament. The current at this point

[7] A. L. Reimann, "Thermionic Emission," John Wiley and Sons, Inc., New York, N. Y., 1934; pp. 227–229.

[8] J. P. Blewett, "Properties of oxide coated cathodes," *Jour. Appl. Phys.*, vol. 10, pp. 668–679; October, 1939.

was used as the total thermionic emission I from the filament independent of field effects.

The log of the total emission in amperes per cm² was plotted against the reciprocal of the absolute temperature (see Fig. 2) to evaluate the emission constant A' and the apparent work function χ of the empirical emission equation (1).

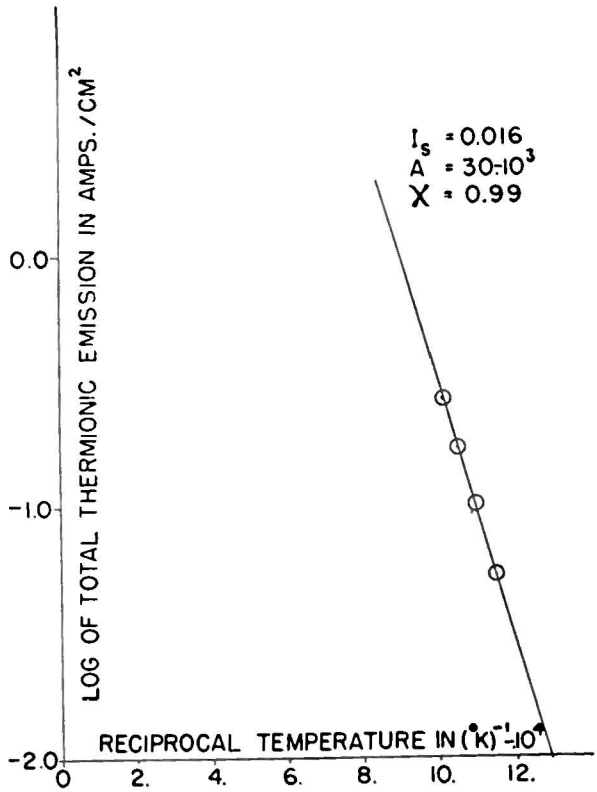

Fig. 2—Plot of emission equation to evaluate A' and χ: $I = A'\epsilon^{-e\chi/kT}$.

It is of interest to note at this point how the Dushman equation can serve just as well to describe the experimental determination of emission as a function of temperature. Using the same data as obtained in Fig. 2 and plotting log I_0/T^2 versus $1/T - 0.10^{-4}$, we find that a fairly good straight line can still be approximated (see Fig. 3). However, the Dushman equation is more complicated in reading, since to get the emission one must multiply by a T^2 factor at any given temperature.

Data

To define the thermionic-emission properties of a given filamentary material, the emission constant A', the work function χ, and the total emission at 800°K I_s were determined as a function of life. The temperature ranges used in the measurements of the above values are also reported. To present this large mass of data in a comprehensive form, a statistical breakdown was employed.

The standard deviation has been selected as a measure of dispersion.

Results are given in Table III.

A particularly interesting feature of the data can now be presented concerning the relationship of A' and χ as a function of life.

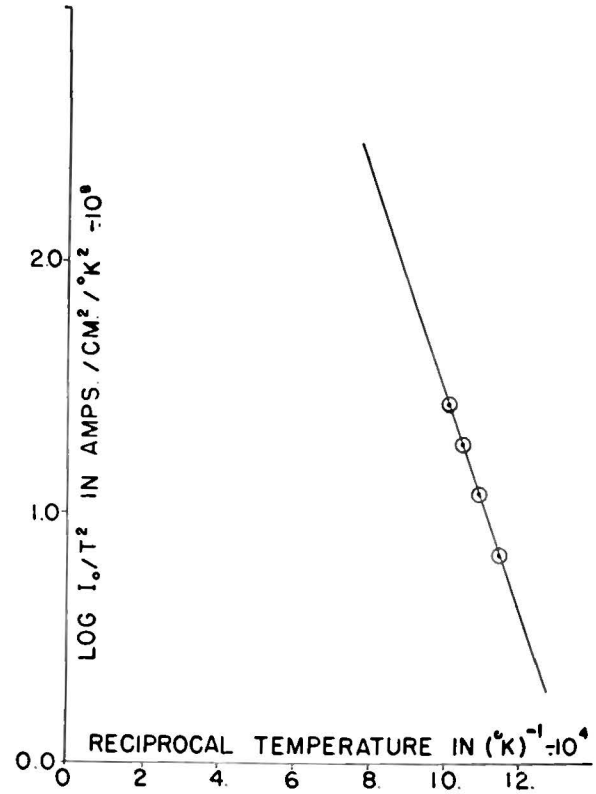

Fig. 3—Plot of Richardson's equation: $I = AT^2\epsilon^{-e\phi/kT}$.

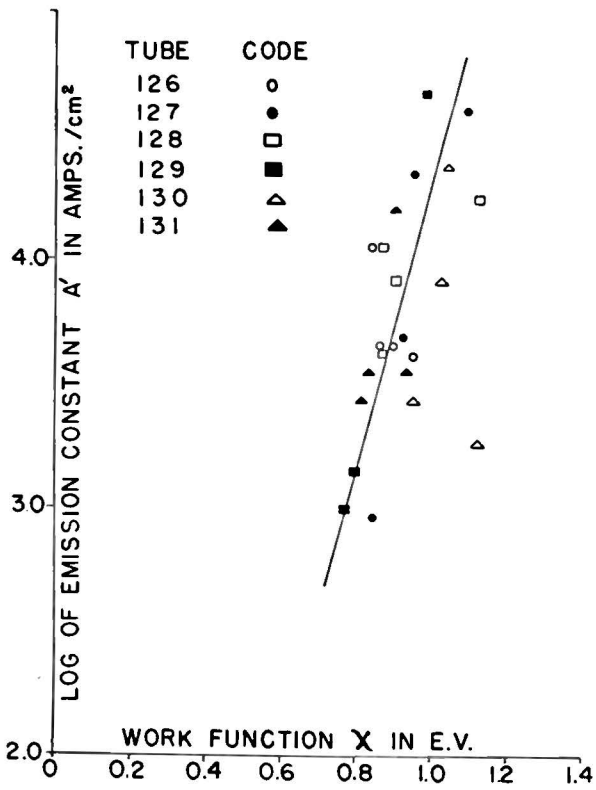

Fig. 4—Variation of A' with χ for pure nickel filamentary alloy.

The measurements of A' and χ were made for each tube at approximately 24, 100, 250, and 500 hours of life. If the tubes were placed in groups such that all of the oxide-coated filaments coated in one type of filament wire were grouped, a straight-line relationship could be found between the log A' and χ. For instance, in Fig. 4 the A' and χ values for six tubes containing a pure nickel filament, coated with the oxide, were studied over life. Each particular reading for this type of material was plotted. The log A' and χ can be seen to be related to each other. Similarly, Fig. 5 represents the data taken for A' and χ of each particular tube at each measurement on life, where the oxide is coated on a nickel wire with boron impurity. Fig. 6 illustrates the data for the commercial Tensite filament coated with the oxide. In the case of all wire samples, as the points of A' and χ for each tube are plotted over various periods of life, a straight-line relation is formed between the log A' and χ. It is also of interest to note that the slopes of these lines are somewhat different, depending on the nature of the core material. It would be of interest to test other core materials and to determine how much the slope will actually be different for different samples. To the authors' knowledge, work of this type has never been published.

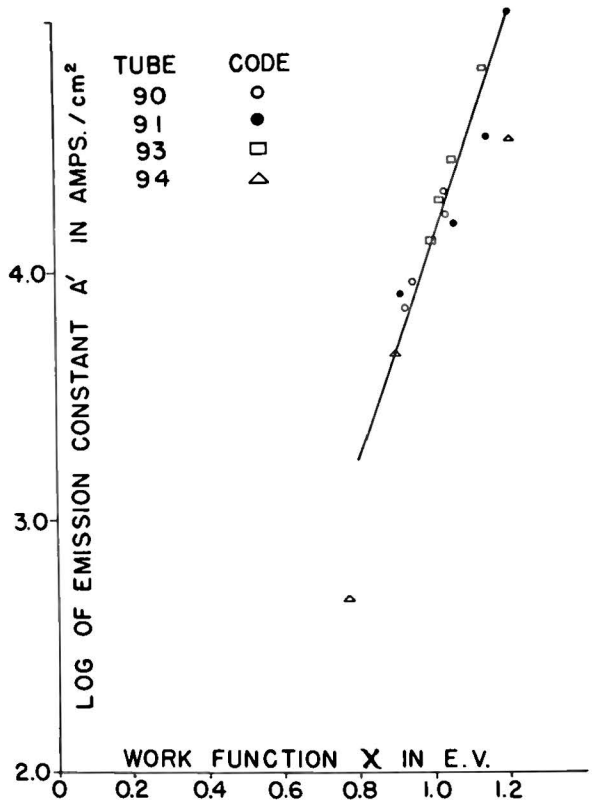

Fig. 5—Variation of log A' with χ for boron-nickel filamentary alloy.

One questionable point is whether this change in A' is a real change in the constant, or whether a change in A' is due to an exponential temperature factor inherent in A'. Suppose, for instance, that in our temperature range of measurement A' changes exponentially with temperature. This could give a result which would mask, to some extent, the nature of log A' varying with χ.

In order to investigate the effect of temperature on A' and the temperature range of the emission measurements, the following device was utilized.

According to Reimann,[9] from photoelectric evidence, a variation in the work function with temperature does not exist.

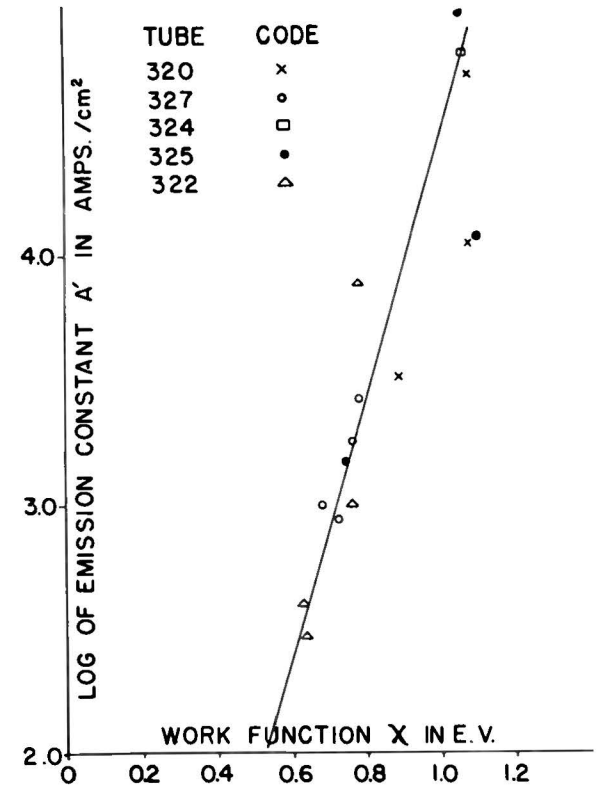

Fig. 6—Variation of A' with χ for Tensite filamentary alloy.

Over the small range in temperatures studied it would not be expected that the work function would shift with temperature. For the case of oxide-coated pure nickel wire, an effective work function of 0.93 electron volts was assumed. From the data in Table III, this is not far from the average value determined.

Then, for each tube, A' was calculated for each I_0 at each particular temperature. This method of determining A' is somewhat independent of the previous method, since here we are assuming a constant work function and calculating A' by a measurement of I_0 as a function of T.

The resulting A' values, as determined from (1), are tabulated in Table IV.

An inspection of these results shows that, if we assume χ to be constant over the range of temperatures in which I_0 was measured, then log A' has relatively small changes in value with temperature, and certainly changes much less than the observed variations in Figs. 4–9.

[9] A. L. B. Reimann, "Thermionic Emission," John Wiley and Sons, Inc., New York, N. Y., 1934, pp. 267–269.

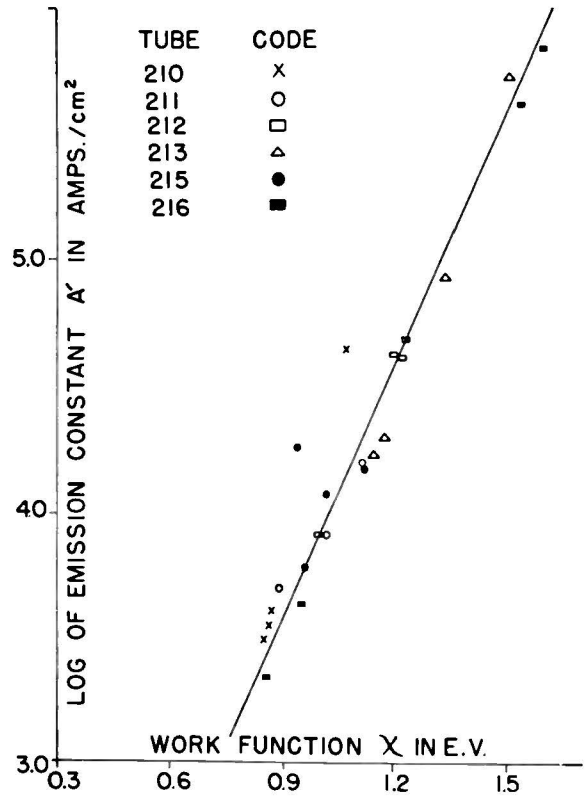

Fig. 7—Variation of A' with χ for vanadium-nickel filamentary alloy.

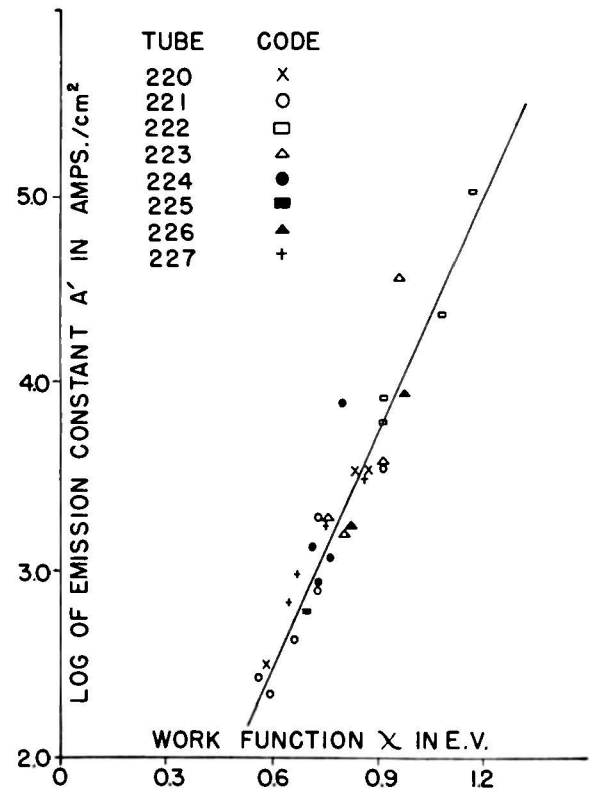

Fig. 8—Variation of A' with χ for cobalt-nickel filamentary alloy.

It might be inferred from this that variations in log A' with χ do not represent a temperature effect but are inherently due to physical and chemical changes in the cathode for small temperature ranges.

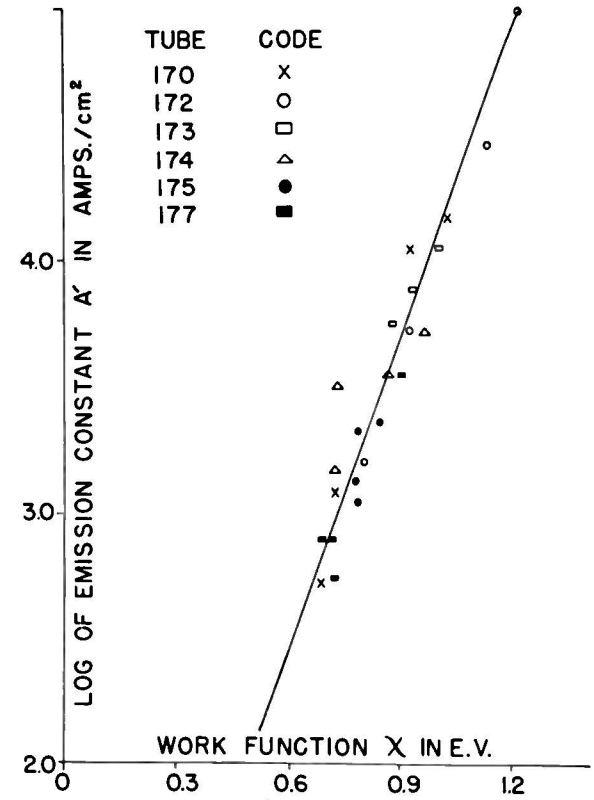

Fig. 9—Variation of A' with χ for platinum-nickel filamentary alloy.

Interpretation

From the data above, it appears that log A' varies linearly with the apparent work function χ, over various periods of life, for the case of oxide-coated cathodes on nickel. Why this should occur is by no means clear, since there is not enough fundamental work yet available upon which to build a general theory. In addition, the results indicate that emission is very much a function of the impurities in the core metal.

TABLE I

1. Bake at 400°C for 30 minutes to drive off occluded gases on glass.
2. Degas elements at red heat not to exceed 850°C by rf heating to thoroughly degas metal parts. (This required about 15 minutes of heating.)
3. Set filament temperature at 900±25°C for 2 minutes (keeping anodes at a red heat) to reduce the alkaline earth carbonates to the oxides.
4. Following this, the processing was established as below:
   (a) Set T_F=1000°C brightness; E_p=10 v for 1 minute
   (b) Set T_F= 900°C brightness; E_p= 50 v for 5 minutes
   (c) Set T_F= 900°C brightness; E_p=150 v for 10 minutes
5. Flash getter and tip off.

TABLE II

Life Test Conditions

T_F=775°C brightness
E_p=40.0 V rms, 60-cycle ac
Emission was measured at 24, 100, 250, and 500 hours of life.

ACKNOWLEDGMENT

Acknowledgment is made for the suggestions offered by W. B. Nottingham, of The Massachusetts Institute of Technology; W. F. Schockley, of the Bell Telephone Laboratories; and George Kuczynski, of Sylvania Electric Products Inc.

TABLE III

MEAN VALUE OF EMISSION CONSTANTS VERSUS LIFE

Alloy Number and Nominal Composition	Hours Life	N	$\overline{A}\cdot 10^{-3}$	$\sigma\overline{A}\cdot 10^{-3}$	$\overline{\chi}$	$\sigma\overline{\chi}$	$\overline{I}_s$	$\sigma\overline{I}_s$	ΔT	T min	T max
1	24	6	21.5	11.9	0.93	0.07	0.029	0.013	130	760	1065
Pure Ni	102	6	9.6	13.3	0.93	0.13	0.017	0.008	135	840	1105
	248	6	4.7	1.6	0.89	0.07	0.013	0.006	145	865	1090
	490	6	4.8	5.6	0.93	0.11	0.009	0.005	120	840	1150
2	25	4	17.1	27.1	0.83	0.09	0.022	0.009	165	815	1100
Tensite	99	4	6.5	5.1	0.83	0.14	0.023	0.022	170	665	1105
1.34% Al, 0.015% Mg, 0.33%	265	4	38.2	36.5	0.90	0.17	0.043	0.028	165	690	1020
Si by weight	501	4	1.53	1.27	0.82	0.15	0.042	0.038	140	620	1080
3	24	4	9.6	5.5	0.94	0.05	0.009	0.010	170	895	1110
B-Ni	102	4	28.8	22.3	1.03	0.16	0.005	0.001	130	940	1115
0.5 Atomic % Boron	248	4	157.0	238.0	1.16	0.18	0.004	0.002	145	925	1190
	490	4	41.0	41.0	1.09	0.12	0.003	0.004	130	925	1175
4	25	3	15.0	8.9	0.98	0.08	0.009	0.005	160	850	1260
Co-Ni	99	3	9.6	1.9	0.92	0.05	0.019	0.015	160	820	1290
2.0 Atomic % Cobalt	265	5	34.4	48.4	0.92	0.125	0.017	0.013	165	815	1265
	501	3	2.9	1.5	0.77	0.014	0.045	0.032	165	735	1250
5	24	6	20.8	35.6	0.91	0.16	0.018	0.019	180	775	1105
Pt-Ni	100	6	3.7	2.5	0.81	0.12	0.017	0.009	180	840	1090
2.0 Atomic % Platinum	250	6	8.8	9.5	0.90	0.14	0.014	0.007	185	830	1095
	506	5	2.8	2.5	0.80	0.03	0.018	0.008	190	830	1105
6	25	6	28.0	39.0	1.03	0.14	0.004	0.003	155	930	1235
V-Ni	100	6	12.8	14.9	0.98	0.15	0.008	0.005	165	840	1190
2.0 Atomic % Vanadium	249	5	15.2	13.6	1.02	0.14	0.007	0.006	150	860	1250
	505	6	225.0	293.0	1.22	0.25	0.007	0.021	140	860	1250

$\overline{A}'$ = arithmetic mean (a.m.) of the emission constant in amp/cm²
$\sigma A'$ = standard deviation (s.d.) of $\overline{A}'$
$\overline{\chi}$ = a.m. of the apparent work function in electron volts
$\sigma\overline{\chi}$ = s.d. of χ
$\overline{I}_s$ = a.m. of total thermionic emission at 800°K true in amp/cm²
$\sigma\overline{I}_s$ = s.d. of $\overline{I}_s$
N = number of tubes
$\overline{\Delta T}$ = a.m. of temperature range used in evaluating A' and χ in °K
T max = maximum temperature used, in °K
T min = minimum temperature used, in °K.

TABLE IV

VARIATION OF LOG A' WITH TEMPERATURE, ASSUMING CONSTANT χ

Tube No.	126		127		128		129		130		131	
No. of Hours	log A'	T	log A'	T	log A'	T	log A'	T	log A'	T	log A'	T
24	4.52	600	4.15	705	4.36	645	4.26	625	3.88	790	4.27	685
	4.56	555	4.26	670	4.39	595	4.28	580	3.85	755	4.37	640
	4.56	520	4.12	630	4.39	560	4.38	540	3.80	765	4.42	600
	4.58	485	4.09	585	4.41	510	4.33	495	3.76	660	4.37	555
102	4.01	725	3.85	775	4.08	700	3.94	770	3.51	830	4.06	735
	4.08	685	3.85	735	4.08	655	3.97	730	3.49	790	4.12	700
	4.05	630	3.79	685	4.06	610	4.03	690	3.44	745	4.15	655
	4.07	580	3.37	625	4.14	565	4.00	640	3.44	700	4.10	605
248	3.92	755	3.70	815	3.55	755	3.92	780	3.53	815	4.09	740
	3.90	705	3.68	770	4.01	695	4.05	730	3.54	780	4.14	695
	3.94	650	3.75	725	4.04	650	4.68	685	3.48	725	4.19	645
	4.17	605	3.71	675	3.97	610	4.11	625	3.48	675	4.21	590
490	3.62	775	3.45	800	3.46	825	3.85	695	3.38	875	3.58	810
	3.54	725	3.43	755	3.40	775	3.88	650	3.46	840	3.57	770
	3.58	675	3.50	700	3.38	725	3.96	605	3.44	805	3.59	730
			3.50	660			3.95	565	3.44	765	3.61	685

T in °C
log A' in amp/cm²