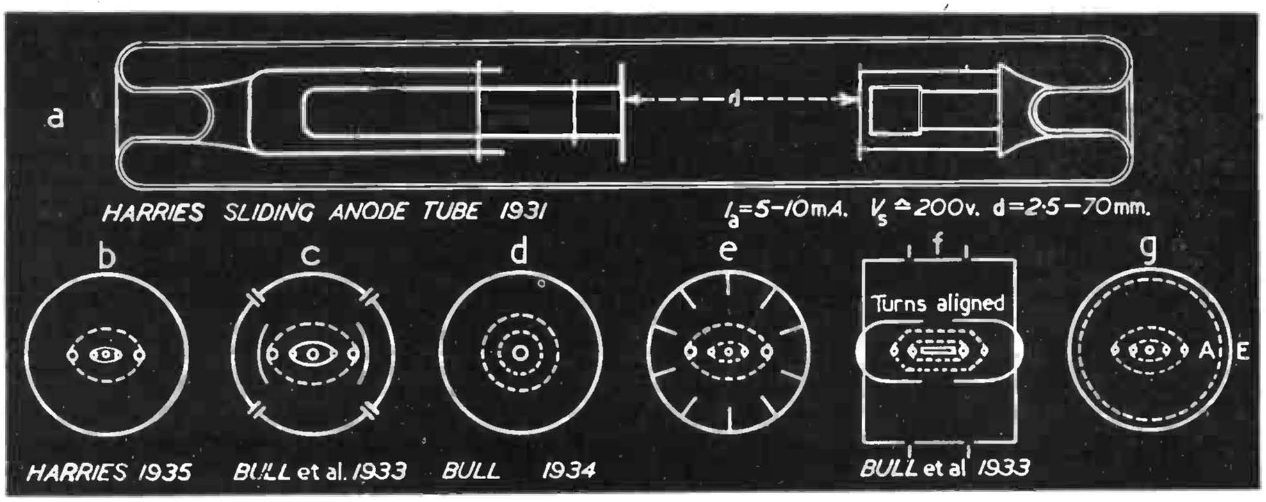

Fig. 2. Various forms of beam tetrode design.

Space Charge and Electron Deflections in Beam Tetrode Theory

By S. RODDA, B.Sc., F.Inst.P.*

1. Introduction

IN this paper the name "beam tetrode" is used to denote a member of the class of valves known variably as output tetrodes, beam tetrodes, kinkless tetrodes or beam power valves.

As in the conventional pentode, the object of beam tetrode design is to prevent the passage of secondary electrons from the anode to the screen when the anode voltage is below the screen voltage, and from the screen to the anode when the anode voltage is greater than the screen voltage. In S.G. valves with low anode voltages, although the full number of primary electrons may be incident on the anode, the loss of secondary electrons from it causes a marked reduction in the net current registered in the anode circuit—indeed the apparent anode current may reverse in sign. The effect gives rise to the well-known "dynatron" I_a, V_a characteristic, and as a result the anode voltage cannot swing below the screen voltage without distortion in the output. In terms of power efficiency this is not serious if the screen voltage can be maintained at a small ratio of the mean anode voltage, but then the mean anode current will be small compared with the anode current obtained by operating at a high screen voltage. It is seen, therefore, that if the passage of retrograde secondary electrons can be avoided very marked advantages will accrue. One way of suppressing the effect would be to find some conducting material for the anode which did not emit secondary electrons when bombarded by primaries, but although the value of γ, the ratio of secondaries emitted per primary, is sometimes small, it is theoretically a forlorn hope to expect γ to be zero. Practically, the situation is worse than this because of the evaporation of barium oxide and of barium from the cathode during the life of the valve, so that any initial surface becomes changed in its secondary emitting properties.

The pentode is characterised by the introduction of another grid, between the screen and the anode; the additional grid, known as the suppressor, is maintained at a low potential and is usually joined directly to the cathode inside the valve.

It is a straightforward solution of the problem and has the additional advantage of providing extra screening between the anode and control grid which is important for high-frequency operation—although the "duplex screen" of the old Mazda AC/SG screened grid valve had the same virtue.

The difference between the S.G. and the pentode or tetrode I_a, V_a characteristics can be seen from Fig. 1. Due to the presence of the low voltage suppressor grid in the pentode, secondary electrons emitted from the anode will have to overcome a voltage drop ΔV before they can enter the suppressor grid to screen space, and it is only the electrons which are emitted with sufficiently high velocities which can do this. This action does not extend down to zero anode voltage, since at some critical anode voltage ΔV vanishes and at lower anode voltages the field between anode and screen directs the secondary electrons back to the screen (*Jonker*).[1] However, the critical voltage is so low that the dynatron kink is entirely removed.

In both tetrodes and pentodes the I_a, V_a curves are characterised by having a rising portion when the anode voltage is increased from zero. At some anode voltage, designated the "knee voltage," the characteristics flatten out more or less abruptly. In order to permit the anode voltages to swing to low values it is desirable that the knee voltage should be as

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MS. received Feb., 1944.

low as possible. In this survey relatively much attention is given to the problem of the steeply rising initial portion of the I_a, V_a characteristic, because of its fundamental physical interest, and in spite of the fact that in normal operation this domain must be excluded from the working range. The anode voltage in the course of its swing will reach low voltages when the control grid is most positive, and in this region the deleterious effects of secondary emission must be removed.

2. Methods of Suppression

In beam tetrodes also the annulment of secondary emission effects is achieved by ensuring in the screen-anode (S.A.) space a sufficient depression in voltage ΔV below the anode. There are two methods which may be employed:—

1. *Space charge suppression.*

The presence of negative charges, *viz.*, the electrons projected into the S.A. space, will reduce the potential at any given point in the space.

2. *Electrostatic field suppression.*

In the parts of the S.A. space not traversed by primary electrons the potential reduction due to space charge will be insufficient; or again, if the density of negative charge in the beam is too small it may become necessary to supplement the voltage depression due to space charge with the voltage depression due to additional electrodes maintained at a voltage lower than the anode, *e.g.*, by an earthed plate system.

We may look on the S.A. space as forming an enclosure, which may contain electrodes to which potentials are applied. Electrons are projected into the enclosure forming regions of negative space charge density. In principle, one can calculate the distribution throughout the given enclosure from a knowledge of

(1) The potentials of all parts of its boundary surfaces and electrode surfaces within the enclosure.

(2) The density of charge at all points throughout the volume.

(3) The Green's function for the enclosure.

The relevant expression for potential V_P at any point P is

$$V_P = \iint V\left(-\frac{\partial G}{\partial n}\right)dS + 4\pi\iiint G\rho d\tau \quad (1)$$

where G, Green's function, is the potential at the volume element $d\tau$ if a positive charge $1/4\pi$ were located at P, with all boundary and electrode surfaces at zero potential.

V is the potential applied at the surface element, $\left(-\frac{\partial G}{\partial n}\right)$ is the normal intensity at the surface due to a charge $1/4\pi$ at P.

ρ is the actual space charge density in the volume.

The first integral in this expression gives the potential due to the electrodes, the second integral gives the potential due to space charge. Since G is positive throughout the enclosure, and since ρ for electrons is negative, there is some voltage depression everywhere due to space charge; but if P is well outside the beam, G will be small inside the beam, and therefore the second term will be small.

Fig. 2 shows various forms of tetrodes which have been proposed. The first tube (a) was described by Harries in Pat. Spec. 385,968, and later in the *Wireless Engineer*.[4] It was a sliding anode tube which was used to demonstrate how the desired tetrode characteristics could be obtained by changing the screen to anode gap; with gaps greater than a certain "critical distance" secondary emission suppression could be obtained. The Harries Hivac tetrode is shown diagrammatically in Fig. 2b (*Wireless World*, 1935).[5]

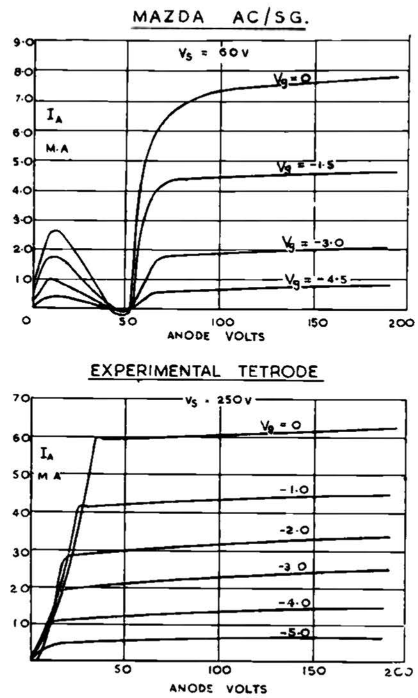

Fig. 1 (a). I_a, V_a **characteristics of S.G. showing secondary emission loss if** $V_a < V_s$.
Fig. 1 (b). Characteristics of tetrode. Pentode usually has more rounded knee.

Two forms of tetrode with earthed side plates proposed by Shoenberg, Bull, and the author[6] are shown in Fig. 2c and 2f. It was recognised that in a valve of normal construction, with grid and screen support rods, it is impossible to avoid regions where space charge is absent. These regions are in line with the supports and may be termed "shadow areas"; as shown above, within the shadow areas the depression of voltage due to space charge in the beams is relatively feeble, so that secondary electrons emitted at oblique angles from the anode are not prevented from traversing these areas and arriving at the screen. By dividing the anode into sectors as shown in Fig. 2c, earthing the side sectors to cathode and also inserting earthed side plates near the screen supports, a relatively low potential is obtained in the shadow areas, sufficient to act as a barrier to secondary electrons.[6,8]

If the field near the suppressor grid wires of a pentode is determined it will be found to be very non-uniform. Now if a voltage depression ΔV is sufficient to inhibit the passage of secondary electrons, is there any advantage in having non-uniform barriers? With regard to the transport of secondary electrons across the barrier, the answer must be that a potential distribution going down to zero voltage must be, if anything, an advantage, but it is offset by the fact that primary electrons which are directed at the suppressor grid wires must themselves be reflected back towards the screen. It was realised therefore that the tetrode construction would eliminate this effect. The direct capture of primary electrons by the screen can be minimised by the correct alignment of screen and control grid wires; therefore in a tetrode with aligned grids the screen to anode current ratio can be reduced to very low values indeed.[33]

In Fig. 2d is shown a cylindrical form of construction suggested by Bull and Keyston.[7] In this valve a "supportless" grid and screen construction is employed; thus the "shadow areas" have been avoided, and it is now no longer logically necessary to introduce earthed electrodes into the screen to anode space. Moreover, by using a cylindrical construction in which the ratio of anode radius to screen radius is fairly large the field strength at the anode becomes small—in the purely electrostatic case the intensity at any point between concentric cylinders is inversely proportional to the radius.

Wing and Young, in the *Proceedings of the Institute of Radio Engineers*, Jan., 1941, have described an interesting U.H.F. high voltage beam tetrode in which an aligned grid and screen structure of vertical slats is utilised, without additional electrostatic suppression.

Fig. 2e shows a slatted anode structure which was an early suggestion for suppressing secondary emission. Even with the anode voltage well below the screen voltage the field within the partial enclosures formed by the anode and its slats is very small; also, a proportion of secondary electrons will be intercepted by the sides of the slats. On the other hand, the field strength along the inner edges of the slats is intense so that secondary electrons will easily leave the edges and reach the screen.

In Fig. 2g is shown an experimental tube, suggested by the author, in which the earthed electrode consists of a cylinder closely surrounding a gridlike anode. If the anode is made positive each anode wire will bear a positive charge, so that the potential diminishes in all directions from the wire. An emitted secondary electron will therefore be attracted back to the anode wire.

3. Concerning Secondary Emission

So far we have accepted the thesis that if in the screen to anode space a voltage depression is established below the anode voltage, then a fairly high fraction of the secondary electrons will be returned to the anode. This statement would cease to be true, for example, if the secondary electrons were projected directly back towards the screen with initial velocities as high on leaving the anode as they possess on striking it—or, in fact, if they have velocities greater than that acquired by an electron in falling through a potential ΔV volts. It has, however, long been known that for small velocities of bombardment a relatively large number of "full velocity" secondary electrons are returned, so that the primary electrons merely appear to be "reflected" without loss of velocity at the target. (Lenard, *Historisches zur Elektronenreflexion.*)[32]

A method for determining the total secondary emission ratio γ is shown in Fig. 3.

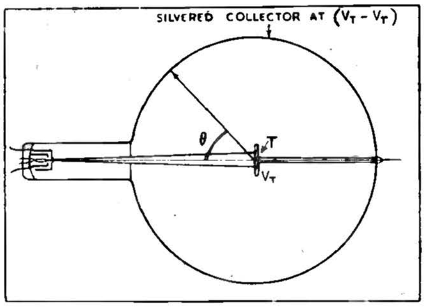

Fig. 3. Method of determining total secondary emission.

A primary beam of narrow divergence is projected at a target T, maintained at a fixed voltage V_T above the cathode potential. A higher positive potential is applied to the spherical collector, so that all the secondary electrons are drawn away and collected. If i_s is the current in the collector and i_T the current measured in the target circuit, then

$$\gamma = \frac{i_s}{i_T + i_s},$$ since i_T is the difference between the incident primary current and the emitted secondary current. The experimental results for γ show the following general features:—

1. The value of γ increases with increasing target voltage, passes through a maximum at voltages of the order of hundreds, and at still higher target voltages it gradually diminishes.

2. For metals, the bombarding voltage at which maximum secondary emission is obtained is approximately proportional to the square root of the density of the metal.

3. Secondary electron emission is greater for oblique angles of incidence on the target. (*Kollath*, 1937; *see* L. M. Myers, *Electron Optics*, p. 304.) These general results are understandable if one supposes that the faster primary electrons penetrate deeper into the target, so that the secondary electrons they liberate from atoms within the metal have a smaller chance of leaving the surface owing to collisions and scattering on the way out. Obliquely incident electrons will not penetrate deeply, but will liberate electrons from near the surface.

4. The value of γ is very susceptible to surface conditions, *e.g.*, adsorbed gas, and for complex layers it may reach high values, over 15.

For nickel anodes in valves γ_{max} is of the order of 1.5—2, and γ_A can be represented approx. by $0.03V_T^{0.7}$ up to 200 volts or so.

In addition to a knowledge of the total secondary emission at a given bombarding voltage V_T, a full description entails a determination of the distribution-in-velocity and the distribution-in-direction of the emitted secondary electrons. A large spherical collector does not discriminate in regard to the angle of ejection; if, therefore, its potential is negative with respect to the target it will collect all the secondary electrons which have a sufficient velocity of emission to overcome the potential drop between the target and the collector. With a plane electrode arrangement, the resolved velocity along the axis must be sufficiently great. In a cylindrical system, with the collector of smaller radius than the target, it can be shown that the collector potential may have to exceed the target potential by a considerable degree before all the secondary electrons are captured. In this case by no means all of the fast retrograde secondaries will be transferred across the potential barrier formed by a potential minimum, but some will return to the anode because of their oblique angles of emission. (Fig. 4.)

4. The Potential Distribution in the Screen to Anode Space in the Absence of Space Charge

If the enclosure forming the screen to anode space is sufficiently long the potential distribution simplifies to a two-dimensional problem, obeying Laplace's equation:—

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0$$

Solutions appropriate to a given electrode arrangement, when the electrode potentials are given, may be determined in several ways:—

1. By calculation for various simple electrode systems.

2. By step-by-step mathematical methods.[9, 10]

3. By using a "rubber drum" method in which a rubber membrane is stretched over a model of the electrode system.[11]

4. By plotting with the electrolytic trough.[12, 13, 14]

As an example of (1) the potential distribution in the cylindrical system depicted in Fig. 2c may be represented by an expansion.

$$V = c + a_0 \log r + \sum_{1}^{\infty}\left(a_n r^n + \frac{b_n}{r_n}\right)\cos n\theta$$

where the coefficients a_n, b_n are found by Fourier analysis. They are such that the boundary conditions around the screen and the anode are satisfied.

By selecting only a few terms in the expression, new equipotential shapes can be determined which will give almost the same potential distribution in the region occupied by the beam.

It is possible to transform any given electrode arrangements in a two-dimensional system by using a conformal transformation. Let x and y be the Cartesian co-ordinates of a point in the first system, and X, Y of the second; then if in general

$$(X + jY) = f(x + jy)$$

where $j = \sqrt{-1}$ the point (X, Y) corresponds to the given point (x, y) and it can be shown that the potential at (X, Y) will be the same as at (x, y) when the corresponding electrodes have the same potentials. If the potential distribution in one system can be found, by any means, the result can be transferred to get the potential distribution in the new system.

5. The Potential Distribution due to Space Charge

When electrons are projected through a screen into the screen to anode space the negative charges produce a potential depression in the space, but the problem of calculating the potential distribution is difficult except for the plane-parallel and cylindrical electrode arrangements with the earthed plates omitted. The potential distribution for the plane-parallel system was obtained by Gill in 1925,[15] and by Bricout[16] in 1926, who also treated the cylindrical case. These results have been amplified by various authors with special reference to beam tetrode systems—for example, by Plato, Kleen and Rothe, and by Salzberg and Haeff.[17-22] The simplest system to consider is the plane-parallel arrangement with a "beam" of infinite lateral extent so that there are no edge effects. Let the screen plane be maintained at an effective potential V_1, the anode at a potential V_2, with the gap x_a cm. (Fig. 5).

Assume that electrons are normally projected with uniform velocity through the screen plane into the screen to anode space.

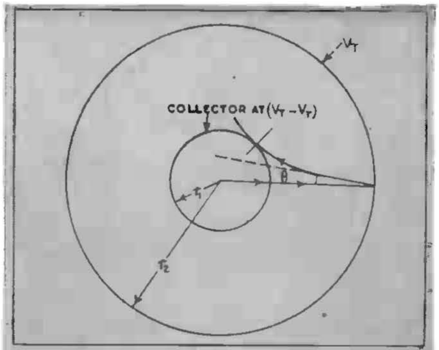

Fig. 4. Collection by a cylindrical collector.

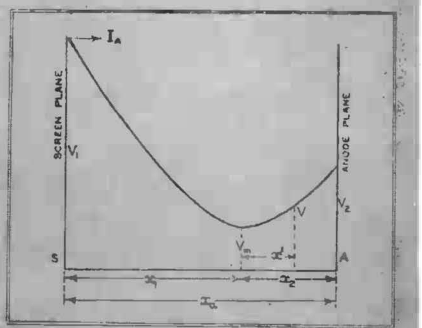

Fig. 5. Potential distribution between screen and anode planes.

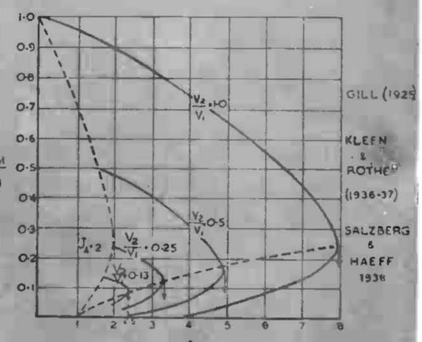

Fig. 6. $\frac{V_m}{V_1}$ as a function of J_a keeping $\frac{V_2}{V_1}$ at fixed values.

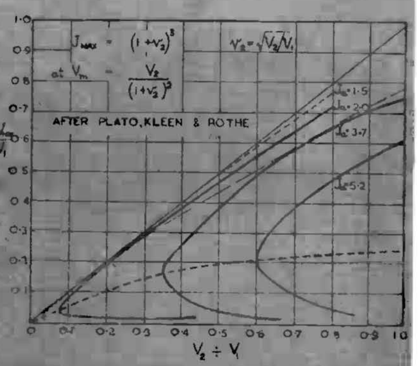

Fig. 7. $\frac{V_m}{V_1}$ as a function of $\frac{V_2}{V_1}$ keeping J_a at fixed values.

The derivation of the potential distribution depends on Poisson's equation

$$\frac{d^2V}{dx^2} = -4\pi\rho \quad \text{.............} \quad (1)$$

ρ is evaluated from the condition that the current density $I_A = u\rho$, where u = the velocity of an electron at the plane where the potential is V.

Now $u = \sqrt{\frac{2e}{m}V}$ if V is the potential above cathode (ESU).

$$\text{Hence } \frac{d^2V}{dx^2} = \frac{4\pi I_A}{\sqrt{\frac{2e}{m}V}} \quad \text{..........} \quad (2)$$

This equation can be integrated, and the conditions imposed that dV/dx = zero at some potential minimum V_m.

Suppose that the origin is taken at the potential minimum position, then the solution can be written

$$J_A^{1/2}\frac{x}{x_a} = (v + 2v_m)(v - v_m)^{1/2} \quad \text{...} \quad (3)$$

where $v = \sqrt{\frac{V}{V_1}}$, $v_m = \sqrt{\frac{V_m}{V_1}}$

and $J_A = \frac{I_A}{I_D}$

I_D is equal to the current density which flows in a plane-parallel diode of gap equal to the screen to anode gap, with its anode potential equal to the tetrode screen voltage. Let the potential minimum to screen distance be x_1 and the potential minimum to anode distance x_2.

$$\text{Then } J_A^{1/2}\frac{x_{1e}}{x_a} = (1 + 2v_m)(1 - v_m)^{1/2} \quad (4)$$

$$J_A^{1/2}\frac{x_2}{x_a} = (v_2 + 2v_m)(v_2 - v_m)^{1/2} \quad (5)$$

so that on summation, since $x_1 + x_2 = x_a$

$$J_A^{1/2} = (1 + 2v_m)(1 - v_m)^{1/2} + (v_2 + 2v_m)(v_2 - v_m)^{1/2} \quad (6)$$

It may happen, however, that the apparent position of the potential minimum lies beyond the anode—this applies when the current density is too low to give a real potential minimum between the screen and anode.

In this case $x_2 - x_1 = x_a$, so that

$$J_A^{1/2} = (1 + 2v_m)(1 - v_m)^{1/2} - (v_2 + 2v_m)(v_2 - v_m)^{1/2} \quad (7)$$

The justification for writing J_A in terms of J_D as the unit of current density appears in the relative simplicity of Equation (6). This unit J_D, which might be called "the diode unit," is in practical measurements equal to

$$\frac{0.234 V_1^{3/2}}{x_a^2} \text{ mA/cm.}^2$$

when V_1 is expressed in volts and x_a in mm. It is important to understand that in practice I_A will be given, so that the valve designer has to adjust I_D to get the appropriate value of J.

Finally, if V_m is ascertained from Equation (6) we can go back to Equation (3) and solve for v explicitly in terms of x. The solution is:—

$$v = \left\{\left(v_m^3 + \frac{J_A x'^2}{2x_a^2}\right) + \sqrt{\frac{J_A x'^2}{x_a^2} + \left(v_m^3 + \frac{J_A x'^2}{4x_a^2}\right)}\right\}^{\frac{1}{3}} + \left\{\left(v_m^3 + \frac{J_A x'^2}{2x_a^2}\right) - \sqrt{\frac{J_A x'^2}{x_a^2}\left(v_m^3 + \frac{J_A x'^2}{4x_a^2}\right)}\right\}^{\frac{1}{3}} - v_m \ldots \quad (8)$$

where x' is the distance from the potential minimum.

In Fig. 6 are shown the values of V_m/V_1 plotted against J_A for various values of V_2/V_1.

Several remarkable conclusions can be drawn from these curves:—

(1) For a given current density there are in certain ranges two possible values of V_m, and so two possible potential distributions, which will allow that current density to be carried in the screen to anode space, *e.g.*, if $J_A = 6.5$ units, the valve can be either 0.5 or 0.1 (in both cases the full current density is transported across the space, so that the two cases cannot be distinguished by readings of anode current). It must be borne in mind that the solutions are steady-state solutions. The conditions for switching from one V_m to another must be examined under transient conditions, in which the instantaneous total current is no longer equal to the electron convection current, but includes the time dependent displacement current.

(2) For given values of V_1, V_2 there is a maximum value of current density which can be transported across the space.

This value is got by differentiating Equation (8) with respect to V_m. The possible J is a maximum when

$$v_m = \frac{v_2}{1 + v_2} \text{ and is equal to } (1 + v_2)^3$$

units.

This raises the question as to what occurs when this maximum current density is exceeded, as although there is this limitation on the current which can be transported, there is clearly no restriction on the current which can be projected into the screen to anode space.

(3) In order to ensure the existence of a potential minimum over the whole range of anode voltage it is necessary that J_A should exceed 2 diode units. The actual value of J_A which just forms a potential minimum is easily found. At low current densities there will be no potential minimum, but the potential will continuously rise from the anode to the screen; then as the current density is increased a potential minimum will appear just at the anode plane, and will just equal the anode voltage. The corresponding value of J_A is obtained by putting $V_m = V_2$ in Equation (8).

In Fig. 7 are shown the values of V_m/V_1 plotted against V_2/V_1 for several constant values of J_A. For comparison purposes a straight line of unit slope is drawn passing through the origin—its ordinate gives the value V_2/V_1, so that the difference of any curve from this straight line is a measure of the voltage depression. For $J_A = 5.2$ the depression is large in value, but the curve does not extend below $V_2/V_1 = 0.6$. For $J_A = 2.0$ the depression vanishes when $V_2/V_1 = 0.25$. For $J_A = 1.5$ (shown in short chain line) there is a slight depression at low values of V_2; over the range $V_2/V_1 = 0.1$ to 0.5 there is no potential minimum at all, while above 0.5 there is voltage depression again.

The long chain line also shown indicates the typical effect produced by earthed side plates, in the absence of space charge. Below some critical anode voltage, lower than the screen voltage, there is no potential minimum; above the critical anode voltage the potential depression becomes increasingly large, so that when $V_2 = V_1$ electrostatic suppression is the important factor, *i.e.*, in the region where the secondary emission loss in the S.G. valve is most severe. At low anode voltages, space charge depression is the more important factor. It should be emphasised that Gill's equations tell us nothing about the shape of the rising portion of the I_a, V_2 characteristic—the theory applies to the full transport conditions, and can only indicate the lowest value of V_2 at which full transport is possible, a value which we can tentatively assume might be the "knee voltage" V_k. If this is so it can be shown that

$$W_k = [J_r^{1/3} - 1]^2 \ldots\ldots\ldots \quad (9)$$

where W_k is the ratio $\dfrac{V_k}{V_1}$

(*To be continued.*)

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