Space Charge and Electron Deflections in Beam Tetrode Theory

Part 3—Conclusion

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Introductory

IN the preceding sections it has been shown that there are two results for knee voltage values, V_K and V_σ, given by Gill's theory and by the virtual cathode theory respectively. The latter value V_σ is much too high and must be rejected.

It has also been shown that the cumulative deflections of electrons at the grid and screen wires may be large, but that then the deflection theory cannot account for sharp "knees."

In the following section a modification of Gill's Equation is proposed, based on a combination of the space charge theory and the deflection theory.

8.1. The Effect of Electron Deflections on the Space Charge and the Current Distribution Between Screen and Anode

There are two cases to be considered:

(*a*) All the current is transported across the S-A space.

(*b*) Part of the current is reflected in the neighbourhood of the potential minimum, the remainder is transmitted to the anode.

8.2. Conditions of Full Current Transported to Anode

The current projected into the S-A space comprised between the angles θ_1 and $\theta_1 + d\theta_1$ is $J_s'(\theta_1)d\theta_1$. At a plane where the potential is V, the forward velocity u is

$$\sqrt{\left(\frac{2e}{m}\right)}\sqrt{V - V\sin^2\theta}$$

and since $V\sin^2\theta = V_1\sin^2\theta_1$

$$u = \left(\frac{2e}{m}\right)^{\frac{1}{2}}\sqrt{V - V_1\sin^2\theta_1}$$

$$\approx \left(\frac{2e}{m}\right)^{\frac{1}{2}}\sqrt{V - V_1\theta_1^2}$$

The contribution $d\rho$ to the total space charge density is $-\frac{dI}{u}$.

To get the total space charge density this must be integrated over all angles.

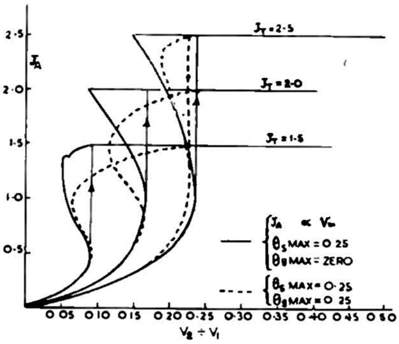

Fig. 14

If the distribution $J_s'(\theta_1)$ is expanded as a series in powers of θ_1, the integration can readily be performed. As an example, if the distribution is uniform from $-\Theta$ to $+\Theta$ we get:

$$\rho = \frac{J_A}{\sqrt{\frac{2e}{m}V_1\Theta^2}}\sin^{-1}\sqrt{\frac{V_1\Theta^2}{V}}$$

As an illustration, consider a potential minimum plane where $V = V_m = V_1\Theta^2$, so that all the electrons are just transmitted.

The space charge density is then $\pi/2$ times as great as if an equal anode current density were transported by undeflected electrons. At higher values of V the multiplying factor will diminish to unity. The overall result is that somewhat less current can be transported than Gill's Equation indicates.

8.3. Current Division Occurs at a Potential Minimum: Part Only Reaching Anode

When the potential minimum is small enough the extremely deflected electrons will not even reach the potential minimum plane, but will be turned back to the screen. Only the electrons for which $V_1\sin^2\theta_1 < V_m$ will reach the anode. The returning electrons will add to the space charge density in the potential minimum to screen space, the more so since their forward and reverse velocities are diminished.

As in the virtual cathode case, the forward current is J_T, the return current is $J_T - J_A$ so that the total current at any plane is $(2J_T - J_A)$.

If for simplicity the effect of electron deflections on space charge is ignored—and to do otherwise makes the calculations very complicated—we can then write as in Equation 4, Section 5.

$$i_1 = i_a \frac{(1 + 2v_m)(1 - v_m)^{1/2}}{\sqrt{2J_T - J_A}} \quad \ldots\ldots\ldots (1)$$

Similarly for the potential minimum to anode gap,

$$i_2 = r_a \frac{(v_2 + 2v_m)(v_2 - v_m)^{1/2}}{\sqrt{J_A}} \quad \ldots\ldots (2)$$

so that on summation,

$$i = \frac{(1 + 2v_m)(1 - v_m)^{1/2}}{\sqrt{2J_T - J_A}} + \frac{(v_2 + 2v_m)(v_2 - v_m)^{1/2}}{\sqrt{J_A}} \quad \ldots (3)$$

Now, if for a given value of J_T the distribution-in-angle of the electrons projected through the screen is known, J_A will be a known function of V_m, denoted by $f(V_m)$. If this is inserted in the equation, it will be seen that it determines V_2.

The results for J_T = 1.5, 2.0, 2.5 are shown as solid lines in Fig. 14, supposing that the entire deflection is at the screen, the maximum deflection being assumed to be 0.25 radian.

As one would expect, the result of giving V_m a positive value is to increase J_A for a given V_2 compared with the virtual cathode case—quite clearly more current ought to be transported across the screen-to-anode space when the electrons are not brought to rest, as at a zero potential minimum. The full current is attained at much lower values of V_2 so that the hysteresis loops are relatively small. Note that if J_A is put equal to J_T, Gill's equation is obtained, while if V_m = 0, the virtual cathode equation is obtained.

The dotted curves in Fig. 14 are drawn on the supposition that there is a maximum deflection at the control grid equal to the maximum deflection at the screen, instead of being zero. The characteristics at low values of I_A are not much altered, and W'_σ is scarcely shifted by this additional deflection. If $V_\sigma < V_1\theta^2$, however, *i.e.*, if the "knee voltage" is lower than the transverse electron velocity, measured in electron volts, the characteristics shoot upwards to current values lower than the full current. (This failure to reach full current may, of course, be accentuated by secondary electron loss from the anode.) If θ_g is approximately equal to θ_s the main effect is an alteration in shape and diminution in area of the loops at high values of I_A. Increasing θ_s to an optimum value greatly reduces the knee voltage to values well below the results given by the virtual cathode theory. Although the knee voltages are now in better accord with experiment, the predicted curves still do not rise quickly enough and in practice the infinitely steep portions are found to be shorter or absent.

The assumption is made in deriving Equation (3) (8.3) that the total current density at any plane lying between the potential minimum and screen is proportional to $(2I_T - I_A)$. This cannot be true near the potential minimum itself since some of the electrons must already have been turned back. In fact, the space charge density has its maximum value somewhere between the screen and the potential minimum, and gradually thins out as the potential minimum position is approached. This is offset by the fact that returning deflected electrons give an excess of space charge, since they move more slowly than deflected electrons across a given plane. For these reasons Equation (3) (8.3) cannot be claimed to be rigorous, but more precise calculations based on specific cases are not in notable disagreement with the knee voltage values given by this simplified theory.

9. Modifying Features

There are, however, several further features that a complete theory should take into account.

(1) *Thermionic Emission Velocities.*

Electrons will be transmitted through the barrier formed by the potential minimum even if this is several tenths of volt negative with respect to the cathode. If $V_1 = 100$ volts and $\theta_1 = 0.04$ radian, $V_t = 0.16$ volt; so that for angular deflections up to this magnitude the transverse ve-

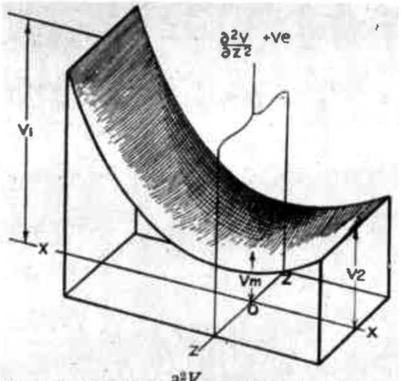

Fig. 15. Showing $\frac{\partial^2 V}{\partial z^2}$ positive in a beam of finite width

locities are comparable with the thermionic velocities. 20 per cent. of the current flow may be comprised within deflection angles from zero to 0.04 radian, consequently at small values of V_m the actual anode current depends on thermionic velocities as well as on the transverse velocities. The result of this is to steepen the initial rising part of the characteristics and to raise the value of I_A at which the curve begins to bend back.

(2) *The Space Charge due to Secondary Electrons.*[6]

Although the potential minimum may present a barrier to the retrograde passage of secondary electrons, the secondary electrons emitted from and returning to the anode constitute negative charges in the potential minimum to anode space. The more copiously secondary electrons are emitted the deeper will be the potential depression they are able to produce. To some degree, therefore, secondary emission by producing space charge exercises a compensating action on the proportion which can travel back to the screen.

(3) *Multiple Trajectories through the Screen Plane.*

Below the knee voltage electrons will be reflected back through the screen, and after reversal in the screen-to-cathode space those which are not intercepted by the screen wires will re-enter the screen-to-anode space. At this traversal these electrons will be deflected, the new angular deflection being either added to or subtracted from the original deflection; in the latter case the electrons may now be able to reach the anode.

(4) *Effect of Finite Width of Beam.*

The results given are strictly applicable only to the plane case for beams of infinitely great cross-sectional area. At the edge of a semi-infinite beam Equation [1(2)] shows that the depression in voltage, due to space charge, is less than half the voltage depression in the interior of the beam. In general, for a beam of finite width, Poisson's Equation is required in its two-dimensional form:

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial z^2} = 4\pi\rho$$

In a section across the beam, V increases from the centre to the edges, $\partial^2 V/\partial z^2$ is positive as in Fig. 15, and hence for a given value of ρ, $\partial^2 V/\partial x^2$ is decreased. This means that even along the centre line of the beam, drawn from screen to anode, the curvature of the V versus x curve is diminished, and therefore for a fixed screen and anode potentials the potential minimum, if formed, will not be so low as with an infinitely wide beam.

(5) *The Effect of Electron Deflections in the S-A Space.*

The electrons moving along the edges of a beam will be repelled by the space charge due to the beam, which may or may not be compensated for by the electrostatic field due to the earthed plate system. In addition to transverse velocities u_y, electrons will acquire transverse velocities u_z, and when

$$u_y^2 + u_z^2 = \frac{2e}{m} V$$

the electrons will be brought to rest in the forward direction.

The simplest case to consider is that of the plane-parallel electrode arrangement with the space charge density insufficient to produce a potential minimum between screen and anode. All electrons will be collected by the anode when $(u_y^2 + u_z^2)$ at the anode plane is less than $\frac{2e}{m} V_2$.

If we suppose that $u_y = ay_0$, $u_z = bz_0$, where y_0, z_0 are co-ordinates giving the point on the cathode from which the electrons are emitted, while a and b are constants, it follows that for an electron to be captured by the anode it must have been emitted within the ellipse given by:

$$a^2 y_0^2 + b^2 z_0^2 = \frac{2e}{m} V_2$$

The area of this ellipse is directly proportional to V_2—hence the emitting area from which the captured electrons originate, and therefore the anode current is initially directly proportional to V_2. If the current density is sufficient to produce a potential minimum, the situation is more complicated, especially when the height of the potential minimum varies across the beam width. At low values of V_2 the current is greater than that calculated on the assumption of an infinitely wide beams—and in general the effect helps to remove hysteresis loops.

Summary

The main points of this survey may be summarised;

1. In beam tetrodes retrograde secondary emission is minimised by producing a sufficient potential depression below the anode voltage by means of space charge and by electrostatic means outside the beam.

2. The equations derived by Gill may be employed to give the potential distribution when a current density, uniform over an infinite plane area, is projected normally through a screen into the screen-to-anode space of a plane-parallel system. The solutions are valid provided that the current density does not exceed a limiting value determined by the screen voltage, the anode voltage and the S-A gap. If the limiting current density is not exceeded the electrons leaving the screen plane are all transported to the anode.

3. Above the critical current density current division must occur somewhere between the screen and anode—some electrons travel to the anode, others are reflected back to the screen. The assumption that this occurs at a virtual cathode does not at all accord with the facts as the calculated knee voltages are much too high, and the predicted enormous hysteresis loops are not obtained.

4. A considerable modification is obtained by supposing that the electrons are given transverse deflections at the grid and screen wires. It is supposed that current reflection occurs at a finite potential instead of at zero potential; on this basis Equation (3) (8.3) is arrived at. The knee voltages are then found to be in reasonable agreement with experiment, while the hysteresis loops are much smaller than on the virtual cathode theory.

5. These results will again be modified because of thermionic velocities, multiple trajectories, etc., and especially because $\partial^2 V/\partial z^2$ is not zero over a beam of finite width.

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Supplementary Note

Equation (3) (8.3) has been put forward on the simplest hypothesis and should only be regarded as a first approximation. It should be emphasised, however, that an exact solution is calculable for any given distribution-in-angle of the current projected into the screen and anode space. In order to carry out the calculation it is preferable to expand the distribution function in powers of $\sin\theta_1$ and then to integrate to find the space charge density, as a function of V in both the screen to potential minimum space, and in the potential minimum to anode space. A first integration with respect to V will give dV/dx and a second integration, which is usually required to be numerical, will give x as a function of V.

The particular case in which the distribution plotted against $\sin\theta_1$ is flat-topped from $-\sin\theta$ to $+\sin\theta$ is the simplest. After calculation this yields as a closely true equation for the characteristic:

$$1 = \frac{(1 + 1.6v_m)(1 - v_m)^{1/2} + (v_T - 1.1v_m)(v_T - v_m)^{1/2}}{\sqrt{2J_T - J_A}} + \frac{(v_2 + 1.6v_m)(v_2 - v_m)^{1/2}}{\sqrt{J_A}}$$

where $v_T = \sin\theta$ and $J_A = \frac{v_m}{v_T} . J_T$

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A New Type of Oscillating Crystal

(*Concluded from p. 648.*)

substance is of the order of ten times the piezo-electric activity of quartz. The crystal in Fig. 1 was cleaved normal to the principal axis, leaving a piece about 2 mm. long. This was found to resonate at the same frequency as before, indicating that there is no longitudinal compression or torsional vibration. On filing to the shape and dimensions in Fig. 2 the frequency of resonance became 340 kc/s. and then as Fig. 3 it became 486 kc/s. These figures agreed (within 5 per cent.) with a law:

$$f = \frac{1,760}{\sqrt{A}}$$

where f is the frequency in kc/s., and A is the cross-sectional area in sq. mm. This would seem to indicate that the vibration is a simple expansion and contraction about the axis.

The indicated temperature-coefficient is -426 parts in one million per $+1°$ C. As this value is more than 200 times the value usually permitted in medium precision quartz, it will be seen that the crystal is quite unsuitable as a frequency stabilising element. While it is possible that a slant-cut crystal might have a zero coefficient, the angle of cutting would need to be very precise in order to obtain a good balance. Further experiments on lithium potassium tartrate crystals indicated that ageing effects are likely to be small, and that the substance is stable at all temperatures likely to be met with in normal apparatus in any part of the world. Breathing upon the crystal damps the oscillations, as with quartz, and it recovers as soon as the moisture has evaporated. While these crystals appear to be useless as frequency stabilising elements the ease with which they can be prepared will be of interest to amateurs. Such crystals might be used as thermometers in a suitable holder, and might also be useful as a stable substitute for Rochelle salt.

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