CORRESPONDENCE

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Power Loss in Electromagnetic Screens

To the Editor, "Wireless Engineer."

SIR,—In the article by Davidson, Looser and Simmonds, appearing in your January issue, the eddy current density in the infinite shielding plane is derived theoretically for a single wire loop, while the measurements were made by using a coil, of length comparable to its distance from the screen, in order to energize this screen.

It is quite easy to extend the method of the authors of the above article in order to obtain an expression for the eddy current density when the energizing factor is a coil, instead of a single loop, and I should like to do so here :

Let the coil be situated so that its axis is perpendicular to the infinite shielding plane and the distance of its middle plane from the shield is a, while its length is $2c$. Using the same notation as in the above-mentioned article and the same units, one can just as easily show that the magnetic potential of the coil is given now by the expression

$$A_\phi' = (\mu/\pi)\, bnI \cos \omega t \int_{a-c}^{a+c}\int_0^{\pi/2} \frac{2\sin^2\theta - 1}{[(b+\rho)^2 + (z-Z)^2 - 4b\rho\sin^2\theta]^{1/2}}\, d\theta dZ$$

where I is the current per turn, n the number of turns per unit length, and Z the longitudinal coordinate of a point situated on the coil.

The magnetic potential of the eddy currents is, then,

$$A_\phi = (\mu/\pi)\, bnI\omega \int_0^\infty\int_{a-c}^{a+c}\int_0^{\pi/2} \frac{(2\sin^2\theta - 1)\sin\omega(t-\tau)}{[(b+\rho)^2 + (Z-z-\frac{2s}{\mu}\tau)^2 - 4b\rho\sin^2\theta]^{1/2}}\, d\theta dZ d\tau$$

and, through the same approximation, as used by Davidson, Looser and Simmonds, we find that, at high frequencies,

$$A_\phi + A_\phi' = -\frac{2bnIs}{\pi\omega}\sin\omega t \int_{a-c}^{a+c}\int_0^{\pi/2} \frac{(Z-z)(2\sin^2\theta - 1)}{[(b+\rho)^2 + (Z-z)^2 - 4b\rho\sin^2\theta]^{3/2}}\, d\theta dZ$$

and, accordingly,

$$i_\phi = \frac{2bnI}{\pi}\cos\omega t \int_{a-c}^{a+c}\int_0^{\pi/2} \frac{Z(2\sin^2\theta - 1)}{[(b+\rho)^2 + Z^2 - 4b\rho\sin^2\theta]^{3/2}}\, d\theta dZ$$

By writing $k^2 = 4b\rho/[(b+\rho)^2 + Z^2]$, we obtain, with the authors,

$$i_\phi = -\frac{nI\cos\omega t}{4\pi b^{1/2}\rho^{3/2}} \int_{a-c}^{a+c} k\left(2K - \frac{2-k^2}{1-k^2}E\right) ZdZ$$

But $ZdZ = -2b\rho d(k^2)/k^4$

Therefore

$$i_\phi = \frac{nI\cos\omega t}{2\pi}\left(\frac{b}{\rho}\right)^{1/2} \int_{Z=a-c}^{Z=a+c} \frac{1}{k^3}\left[2K - (1 + \frac{1}{1-k^2})E\right] d(k^2)$$

Referring, now, to the integral formulae for complete elliptic constants, as given by Jahnke and Emde : "Tables of Functions" (Dover Publications —New York 1943) p. 78, and writing, also,

$$u^2 = 4b\rho/[(b+\rho)^2 + (a+c)^2]$$

and $v^2 = 4b\rho/[(b+\rho)^2 + (a-c)^2]$

we obtain, finally,

$$i_\phi = \frac{nI\cos\omega t}{\pi}\left(\frac{b}{\rho}\right)^{1/2}\left\{(1/u)\,[2E_u - (2-u^2)K_u] - (1/v)\,[2E_v - (2-v^2)K_v]\right\}$$

the subscripts denoting the moduli of K and E.

We may still simplify this expression by using the notation of Jahnke and Emde, and, actually,

$$i_\phi = \frac{nI\cos\omega t}{\pi}\left(\frac{b}{\rho}\right)^{1/2}(v^3C_v - u^3C_u)$$

The complete elliptic integral C appearing in the above final expression for the eddy current density is tabulated for intervals of 0.01 of k^2 on p. 82 of the above edition of Jahnke & Emde.

C. A. SIOCOS.

Stanford University,
California, U.S.A.

Beam Tetrodes

To the Editor, "Wireless Engineer"

SIR,—It is generally recognised that when the primary current density flowing into the screen-to-anode space falls below a limiting value, it is not then always possible to ensure the formation of a potential minimum in this space by the charges due to primary electrons alone ; this situation is especially true when the anode voltage is approximately one-quarter of the screen voltage. It has consequently been suggested by several writers that the space charge arising from secondary electrons plays an important role in producing a potential minimum when the space charge due to primary electrons is inadequate. The computation of the magnitude of the effect is fairly difficult, and can only be performed if there is sufficient data regarding the amount of the possible emission and the velocity distribution.

A great simplification can be obtained, however, if we may postulate that the secondary electrons can be emitted copiously with zero velocities of emission ; we then have the condition that the secondary electron flow is limited to the value which sets up just enough additional space charge in the screen-to-anode space to reduce (dV/dx) at the anode to zero.

If this assumption is made the equation giving the potential distribution in the screen to anode space is

$$\frac{d^2V}{dx^2} = \frac{4\pi}{\sqrt{2\epsilon/m}}\left[\frac{I_P}{\sqrt{V}} + \frac{I_S}{\sqrt{V - V_2}}\right]$$

Where I_S is the secondary current density
I_P is the primary current density.

If I_S/I_P is denoted by γ, it can be shown that the following equation serves to determine γ:—

$$\sqrt{J_P} = \int_0^{(1-W_2)^{3/4}} \frac{dZ}{\sqrt{\frac{\sqrt{W_2 + Z^{4/3}} - W_2^{1/2}}{Z^{2/3}} + \gamma}}$$

Where

$$J_P = \frac{I_P}{I_D};\ I_D = \frac{\sqrt{2\epsilon/m}}{9\pi}\ \frac{V_1^{3/2}}{x_a^2};\ W_2 = \frac{V_2}{V_1}$$

V_1 = screen voltage
V_2 = anode voltage
x_a = screen-to-anode gap.

I_D is, of course, the current density in a plane diode of anode voltage V_1 and gap x_a. The results show that for $J_P = 1$, γ does not exceed 0.2, even when $W_2 = \frac{1}{4}$. $J_P = 1$ may be considered to correspond to a current density less than half the peak current density, and the analysis confirms that the space charge due to secondary electrons does in fact considerably help to suppress the retrograde flow of secondary electrons.

S. RODDA.

Enfield, Middx.

Spectrum of a Phase- or Frequency-Modulated Wave

To the Editor, "Wireless Engineer"

SIR,—In a paper in *Wireless Engineer* for March 1944, Mr. F. M. Colebrook pointed out some interesting features of the spectrum of a wave sinusoidally modulated in phase or frequency. The object of the present note is to generalize these results somewhat and in particular to consider the case in which the carrier frequency is an integral or half-integral multiple of the modulation frequency. In this case each sidewave is a doublet and in general the mean square value of the wave is not equal to half the square of the amplitude.

A general representation of a wave of unit amplitude sinusoidally modulated in phase or frequency is

$$z = x + jy = \exp j\,[\omega_0 t + m \sin (pt + \phi)] \quad (1)$$

and by taking the real or imaginary part of z and by giving ϕ any value the complete range of cases can be considered.

The wave can be expanded into the spectrum

$$z = \exp (j\omega_0 t) \sum_{n=-\infty}^{\infty} J_n(m) \exp j^n(pt + \phi) \quad \ldots \quad (2)$$

giving
$$x = \cos [\omega_0 t + m \sin (pt + \phi)] = \sum J_n \cos [(\omega_0 + np)t + n\phi] \quad \ldots \quad (3)$$

$$y = \sin [\omega_0 t + m \sin (pt + \phi)] = \sum J_n \sin [(\omega_0 + np)t + n\phi] \quad \ldots \quad (4)$$

Since n assumes all values from $-\infty$ to $+\infty$ it is seen that when n is negative and greater in magnitude than ω_0/p the frequency terms become negative but with the identities

$$\cos (-qt + n\phi) \equiv \cos (qt - n\phi)$$
$$\sin (-qt + n\phi) \equiv -\sin (qt - n\phi)$$

are transformed into the corresponding positive frequencies. This implies that there are two series of spectral lines:

(i) with $n \geqslant -\frac{\omega_0}{p}$ and (ii) with $n < -\frac{\omega_0}{p}$

which for convenience may be termed the positive series and the negative series respectively.

A special case is that in which the two series coincide in frequency which occurs when $2\omega_0/p$ is either an integer or a half-integer. If $2\omega_0/p = N$ it is seen that the nth and the $-(N + n)$th side-waves coincide and have the frequency

$$\omega = \omega_0 + np = -[\omega_0 - (N + n)p] \quad \ldots \quad (5)$$

when N is even the lowest frequency is zero and corresponds to the single $(-\frac{1}{2}N\,\text{th})$ component; when n is odd the lowest frequency is $p/2$ and is a doublet corresponding to the $-\frac{N-1}{2}$th and $-\frac{N+1}{2}$th components.

The spectrum (2) may then be written

$$z = \sum_{n_0}^{\infty} \{J_n \exp j(\omega t + n\phi) + J_{-N-n} \exp [-j(\omega t + \overline{N + n}\,\phi)]\} \quad \ldots \quad (6)$$

where ω is related to n by equation (5) and n_0 is $-\frac{N-1}{2}$ when N is odd and $-N/2$ when N is even except that only *one* term of zero frequency is to be included in the summation.

The coincidence of the two series of sidewaves affects the calculation of the mean square value of the wave. When they are not coincident the mean square value is as Colebrook showed

$$\overline{x^2} = \overline{y^2} = \tfrac{1}{2} \sum_{-\infty}^{\infty} J_n^2 = \tfrac{1}{2} \quad \ldots \quad (7)$$

In the case of coincidence the mean square value may be greater or less than $\frac{1}{2}$ depending upon the values of N and ϕ. The value may be found either by summing the squares of the resultant side-wave amplitudes given by equation (6) or, more directly, averaging by integration over a period $2\pi/p$ of the modulation frequency. Since

$$\overline{x^2} + \overline{y^2} = |z|^2 = 1$$

it is only necessary to find x^2, and y^2 is then also determined. Now

$$\overline{x^2} = \frac{1}{2\pi}\int_0^{2\pi} \frac{1 + \cos [2Npt + 2m \sin (pt + \phi)]}{2}\, d(pt)$$
$$= \tfrac{1}{2}[1 + (-)^N J_N (2m) \cos N\phi] \quad \ldots \quad (8)$$

by well-known integral formulae. The fractional departure of $\overline{x^2}$ or $\overline{y^2}$ from $\frac{1}{2}$ cannot exceed $J_N (2m)$ and in practice this will be an extremely small quantity since $\omega_0 (= \frac{1}{2}Np)$ is always large compared with the maximum frequency excursion $\delta\omega$ $(= mp)$.