TABLE I

	Calculated	Measured
f_r	507.797 kc.	508.090 kc.
L_k	0.981 h.	1.036 h.
C_k	0.100 $\mu\mu$fd.	0.095 $\mu\mu$fd.
C_p	13.5 $\mu\mu$fd.	16.8 $\mu\mu$fd. (in holder)

The ratio C_p/C_k is about 175. The temperature coefficient was measured at about $25\times10^{-6}/°\mathrm{C}$.

If the same electrical constants were to be obtained with a crystal plate vibrating in the fundamental mode and using the more normal ratio $l_{z'}/l_{y'}=0.7$, the dimensions would become $l_z=0.05$ mm., $l_{y'}=5.22$ mm., and $l_{z'}=3.65$ mm., which cannot be realized in practice.

Positive-Grid Characteristics of a Triode[*]

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Summary—**By operating the grid of a triode at a positive potential it becomes a virtual cathode discharging electrons with uniform initial velocity, and thus a recent theory by Jaffé becomes applicable. The positive-grid characteristics of a number of radio receiving tubes are determined and matched with those determined from theory. Agreement, satisfactory under given conditions, is found in the region of low plate voltages.**

Departure of the experimental curves from the theoretical ones is discussed. Practical tubes tested do not, by any means, correspond to the ideal conditions of the theory: the electrodes are not plane; the electrostatic field is not homogeneous; secondary emission from the grid is present; contact potentials exist; and there is always a velocity distribution among the electrons. Nevertheless, the investigation points a way toward a better theory of the triode.

INTRODUCTION

THEORY CONCERNING the triode is meager, the familiar Van der Bijl[1] equation being determined experimentally and holding over the straight portion of the tube characteristic and for negative grid potentials only. In a recent paper, Jaffé[2] treated the case of the diode in which the electrons have uniform initial velocity. This theory would seem to be applicable to the triode under certain conditions. It was the purpose of this investigation to test the validity of the Jaffé theory under those conditions, which will be discussed later.

THE JAFFÉ THEORY AND ITS PRESENT APPLICATION

The Jaffé equation is derived for the case where electrons enter, at right angles and with uniform velocity, the space between two infinite parallel planes. The equation is

$$j = V_0^{3/2}\left[\left(1+\frac{E_0}{V_0}\right)^{1/2}+\left(\frac{E_0}{V_0}\right)^{1/2}\right]^3 = [(E_0+V_0)^{1/2}+E_0^{1/2}]^3 \qquad (1)$$

where j is introduced as the "reduced current density" by the relation

$$j = \frac{9\pi i d^2}{\left(2\dfrac{e}{m}\right)^{1/2}}. \qquad (2)$$

Here

i = current density

d = distance between emitter plane and receiver plane

e = the charge of the electron

m = the mass of the electron

E_0 = the potential characteristic of the initial electron velocity

V_0 = the potential difference (not necessarily positive) between the emitter plane and the receiver plane.

For a given vacuum tube, the "reduced current density" j is equivalent to the true current density i multiplied by a constant factor, depending upon d. It is this factor which permits the reduction of the formula to one depending upon a single parameter only, i.e., E_0/V_0. The formula holds for that part of the characteristic which is space-charge-limited. As soon as the current reaches the maximum $j_{\max}$ given by the supply of the cathode (temperature saturation), the characteristic automatically assumes a constant value. It is this feature, along with the fact that infinite values of space-charge density do not appear, which constitute the main improvements of the Jaffé formula over the Child-Langmuir formula, into which it degenerates in the limiting case $(E_0/V_0)\rightarrow 0$.

In the present paper, the application of the Jaffé theory must be limited to positive grid potentials, for in this manner ideally it should be possible to impart to the electrons, which are emitted from the cathode with their thermal velocities, a uniform velocity. The electrons are accelerated in the field of the positive grid, between the cathode and the grid, and, after passing through the grid meshes, have velocities which differ only in the amounts of their initial velocity—now a very small percentage of difference. The grid becomes, there-

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[1] H. J. Van der Bijl, "Theory of the thermionic amplifier," *Phys. Rev.*, vol. 12, pp. 171–198; September, 1918.

[2] George Jaffé, "On the currents carried by electrons of uniform initial velocity," *Phys. Rev.*, vol. 65, pp. 91–98; February, 1944.

fore, a virtual cathode, discharging into the grid-plate space electrons of nearly uniform velocities, and the tube is then treated as a diode according to the Jaffé equation.

The E_0 of this equation, which is the *e*-volt velocity of the electrons, then becomes the customary E_c of vacuum-tube theory, while the V_0 of the equation becomes the plate potential measured with respect to the grid—the virtual cathode. The following relationship is evident: $E_b = E_c + V_0$. It is because of this relationship that a new method of plotting plate characteristics is presented. In this method, V_0 is chosen as abcissa and E_0 as parameter, and the characteristic extends to negative values of V_0 with the limit $V_0 = -E_0$. The new method is used throughout this paper, and results in a better definition between curves at low plate potentials.

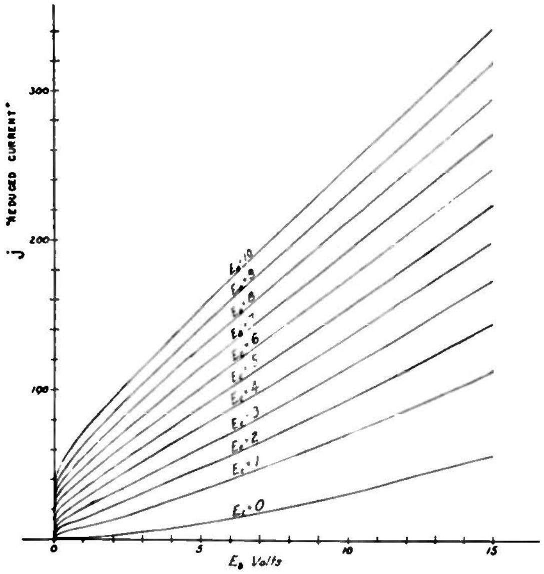

Fig. 1—Theoretical plate characteristics for low voltages, plotted in the usual way.

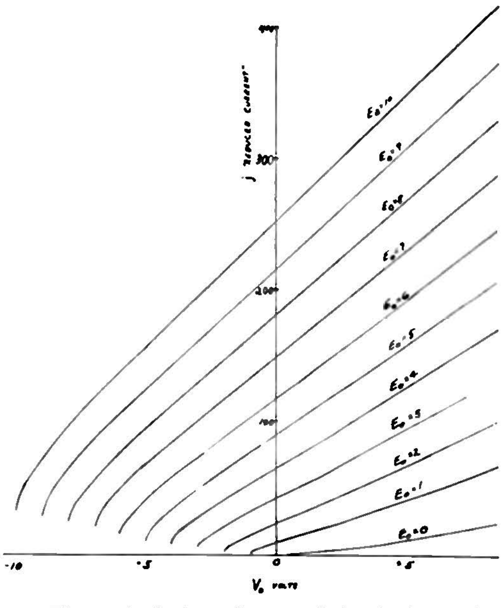

Fig. 2—Theoretical plate characteristics for low voltages, plotted by the new method.

Since experimental agreement with the Jaffé theory was sought for in conventional radio receiving tubes, which by no means correspond to the ideal conditions of the theory, it soon became evident, for reasons which will be discussed later, that such agreement would most likely be found in the region of low plate potentials. Figs. 1 and 2 show the theoretical plate characteristics plotted in the conventional way and in the new way.

It should be noted that the theoretical plate characteristics start with a finite value and a vertical tangent for all positive grid potentials. If there were no space-charge effect, the current would be equal to j_{max} even for the limiting value $V_0 = -E_0$. Because of the space-charge effect, however, this value is reduced to $E_0^{3/2}$. In practical cases there will always exist a velocity distribution with velocities ranging from zero to a maximum value, and, as might be anticipated, the characteristics will not start with a finite value of the current at $V_0 = -E_0$, but will descend to zero.

EXPERIMENTAL MEASUREMENTS

The experimental arrangement was identical with that used in determining the static characteristics of vacuum tubes, as can be seen from the schematic diagram, Fig. 3. Certain precautions were observed in the taking of data. If the grid was operated at too high a positive potential while the plate voltage was low or zero, there was the possibility (and sometimes actuality) of the grid being heated by its own current and becoming an actual emitter of electrons. This was one reason for performing the experimental work in the region of low grid and plate potentials. At all times cathode current was limited to a value that was considered safe, although in practically all test runs the specified maximum for the plate current was exceeded.

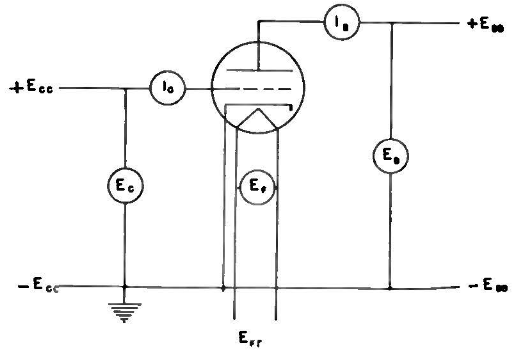

Fig. 3—Schematic diagram of the experimental arrangement.

Tubes for the determination of positive-grid characteristics were selected from those readily available as radio receiving tubes. While it was found that there are many triodes available, a large majority of the various types are duplicates, with the exception of filament voltage and current requirements. The selection was thus narrowed down to such representative types as the following: 45, 26, 2A3, 6J5GT, and 76. In order that additional data might be secured, several tubes of more than three elements were tried, with the additional elements so connected externally as to cause the tube to act as a triode. In this line of investigation the type-6AG7 tube, with the suppressor grid and screen grid connected to the plate, was found to be most satisfactory.

DISCUSSION AND COMPARISON BETWEEN THE THEORY AND EXPERIMENTAL RESULTS

In attempting to fit characteristics observed in commercial tubes with those determined from the theoretical formula which holds under highly idealized conditions, it must be remembered that there are many circumstances which might be expected to prevent an accurate agreement. The most important circumstances are:

(a) The electrons in the actual tubes will not have really uniform velocities, even under the conditions previously outlined.

(b) The field in the neighborhood of the grid (which is acting as a virtual cathode) will be very far from homogeneous.

(c) The electrons will therefore leave the plane which idealizes the grid, not only in normal directions.

(d) Since some of the lines of force which emerge from the real cathode terminate on the grid, electrons will be dragged into the grid beyond the number of those which are prevented by the space charge from entering the space between the grid and plate. The number of electrons thus lost will depend greatly on the size of the grid meshes and on the values of E_0 and V_0.

(e) Contact potentials may and do occur in the tubes and will affect the experimental curves strongly, particularly in the domain of low values of E_0 and V_0.

(f) Since the grid is attracting electrons by its positive potential, it is quite likely that secondary electrons will be liberated, and will thus add to the plate current. It should also be observed that, in adapting the experimental curves to the theoretical ones, only *one* constant is available—that which reduces the observed currents to the "reduced current density" of the theory. This one factor reduces the whole set of curves, and therefore any agreement obtained offers a very severe test of the theory.

Under these circumstances, the best to be expected is semiquantitative agreement between the theoretical and experimental curves. A close examination shows this to be true. Comparing the behavior of the experimental and theoretical curves (see Figs. 2, 4, and 5), a complete qualitative agreement can be observed; the characteristics begin approximately at those negative values of V_0 which reduce the initial velocities of the electrons to zero, cut the zero-voltage axis at positive values of plate current I_b which increase with the initial velocity given the electrons, and finally become very nearly parallel for all velocities.

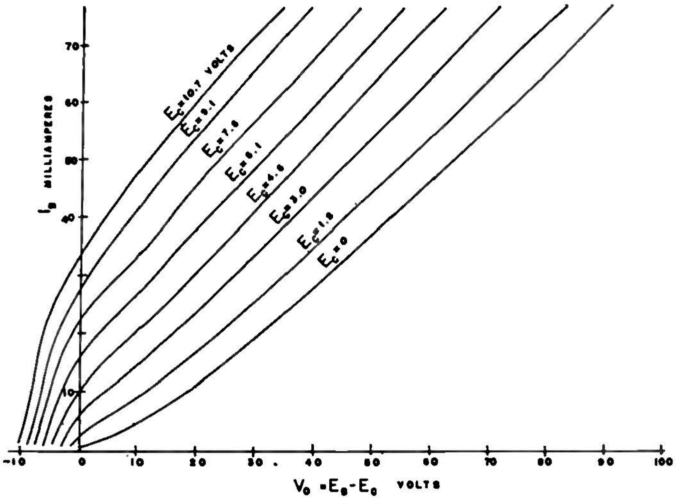

Fig. 4—Experimental plate characteristics for a type 2A3 tube.

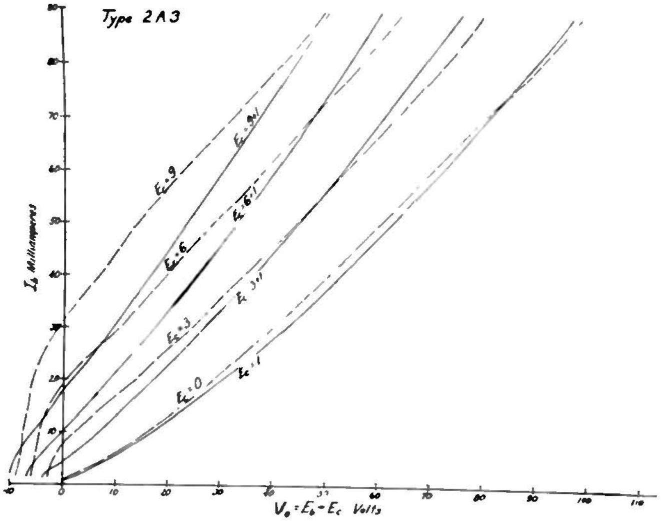

Fig. 5—Experimental and theoretical plate characteristics plotted over a wide range of plate voltage, for a type 2A3 tube. The theoretical curves have been multiplied by a factor of 0.070, and a contact potential of +1 volt on the grid has been assumed.

According to theory, the currents observed for $V_0=0$ (i.e., $E_b=-E_c$) should be given by $j_{V_0=0}=8E_0^{3/2}$ and in a particular tube be proportional to $E_0^{3/2}$ (or to the third power of the initial velocity). This consequence of the theory is verified with fair accuracy, as is shown in Table I. Deviations are strongest in those cases where secondary emission is strong, as we will see below.

Attempts to obtain numerical agreement over wide ranges of potentials met with serious difficulties in all cases. These difficulties begin with the ordinary Child-Langmuir characteristics, i.e., for $E_0=0$. It is well known that in the relation between current and plate voltage for diodes (or triodes used as diodes), the exponent almost never shows the theoretical value 3/2,

TABLE I

THE DEPENDENCE OF THE CURRENT ON THE GRID POTENTIAL FOR $V_0=0$

Tube Type	E_0	j	$E_0^{3/2}$	$\dfrac{j}{E_0^{3/2}}$
26	3	2.25	5.196	0.434
	6	5.0	14.697	0.340
	9	8.85	27.000	0.328
45	3	3.3	5.196	0.635
	6	7.55	14.697	0.515
	9	12.25	27.000	0.454
76	3	3.7	5.196	0.635
	6	7.1	14.697	0.483
	9	12.5	27.000	0.463
2A3	3	7.9	5.196	1.52
	6	18.5	14.697	1.26
	9	32.5	27.000	1.24
6AG7	3	9.9	5.196	1.91
	6	25.05	14.697	1.71
	9	48.0	27.000	1.78
6J5GT	3	3.25	5.196	0.625
	6	8.4	14.697	0.572
	9	16.0	27.000	0.593

and furthermore changes with plate potential. The same might be expected if a triode is operated with $E_0=0$, since this arrangement corresponds to the assumption made in Child and Langmuir's theory; namely, that the electrons leave the cathode with no velocity. Measurements made confirmed these observations. In consequence of these circumstances, the curves for $E_0=0$ cannot be considered standard curves for the calculation of the reduction factor. Rather, this factor has to be determined in a way which reduces the whole set of curves for best agreement. Since the currents for $V_0=0$ show the necessary theoretical dependence on E_0 (as indicated above), the reduction was performed in such a way that these currents were made to coincide approximately for the experimental and theoretical sets.

Fig. 5 shows the theoretical and experimental curves plotted over a wide range of plate voltage. The attempt is made to match the two sets of curves as nearly as possible; however, the initial hump in the experimental curves prevents any close agreement, in this wide range. The same difficulty was encountered in all of the tube types tested. In the calculation of the theoretical curves represented in this figure, a contact potential of +1 volt on the grid had to be assumed in order to make the $E_0=0$ curve fit.

It was later found that, by matching the experimental and theoretical curves in the range of V_0 below 20 volts, much closer agreement could be obtained. Fig. 6 illustrates such agreement. Experimental points were obtained (as indicated in the figure) with tube types 2A3, 76, 6J5GT, 45, and 26. Since all triode plate characteristics are very similar, they can be brought into near coincidence by multiplication by a constant factor which depends upon the construction of the tube and the temperature of the cathode, but which was chosen experimentally so as to give the best agreement between curves in the region in question.

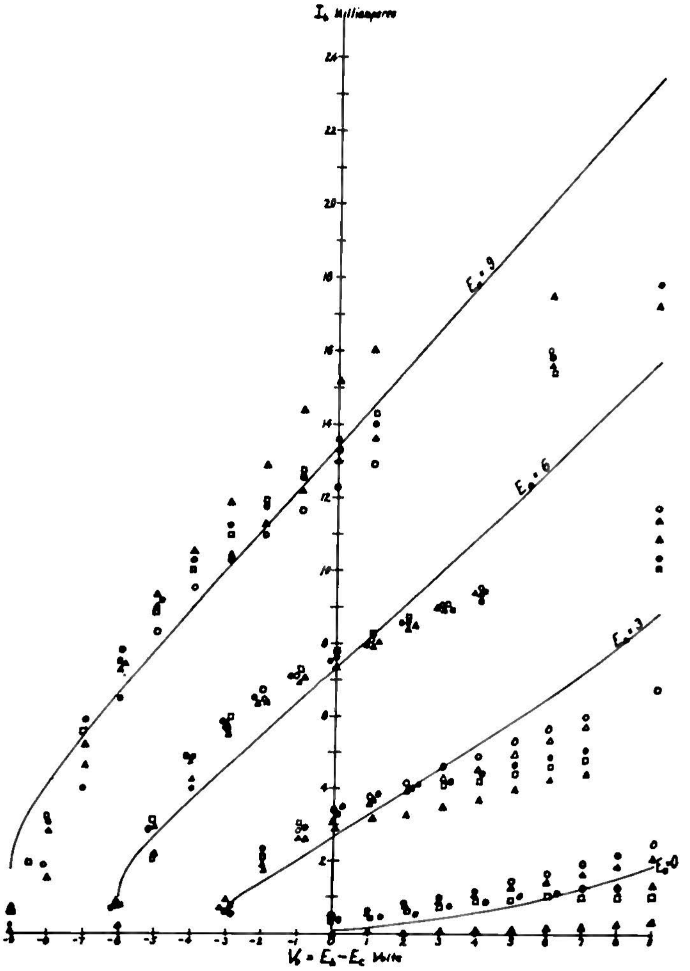

Fig. 6—Comparison between theoretical and experimental characteristics for 6J5GT, 26, 76, 2A3, and 45 tubes, using low plate voltages. Here the theoretical values have been multiplied by 0.062 to bring them to milliamperes.

Key:
○ = Type 45 (not reduced)
△ = Type 2A3 (×.4)
□ = Type 56 (×1.2)
▲ = Type 6J5GT (×.9)
● = Type 26 (×1.5)

A type 6AG7 pentode was caused to act like a triode by connecting the suppressor grid and the screen grid to the plate, and plate characteristics were determined. It was not possible to secure good agreement between theory and experiment over a wide range of values, but in the region of V_0 below 20 volts good agreement is obtained, as is shown in Fig. 7.

As for the systematic deviations of the theoretical curves from the experimental ones, inspection of Figs. 3 through 7 will confirm the following observations:

(a) In all cases, even in the low-voltage region, points determined experimentally fell below the theoretical curves for the first few volts of rise of plate voltage. This depression increased with increasing grid potentials, and was obviously due to the grid attracting electrons which would otherwise flow to the plate. Depending upon the construction of the grid and its potential, the grid current became quite large.

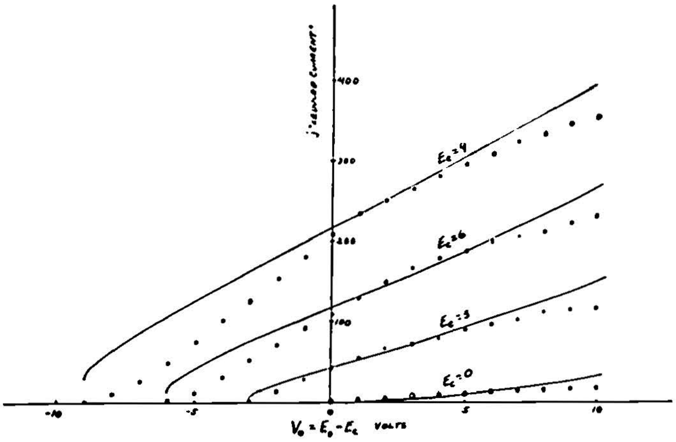

Fig. 7—Comparison between theoretical and experimental plate characteristics for low plate voltages, type 6AG7 tube. The experimental values (O) have been multiplied by a factor of 4.33.

(b) A pronounced hump, following the initial depression, is found in all of the experimental curves. This is quite probably due to secondary emission from the grid. Such emission would not be evident when the grid potential is zero or negative (few electrons strike the grid), nor to any degree while $V_0 < E_0$ (the secondary electrons return to the grid); but for $V_0 \geqq E_0$ most of the secondary electrons released would flow to the plate, adding to the plate current.[3,4] This would account for the hump in the characteristics very well. The contribution to plate current by these electrons can only be guessed, but comparison between this hump and the depression due to secondary emission from the plate of, and found in the plate characteristics of, the old type 24-A tetrode (which was noted for this effect) is suggested.

(c) The theoretical curves invariably rise above the experimental ones for voltages above $V_0 = 20$ volts. This deviation is rather hard to account for. It can hardly be attributed to deviation from the 3/2-power law. It is quite possible that the magnitude and extension of the secondary-emission effect noted above is greater than originally anticipated.

(d) The Jaffé equation is derived for parallel planes. Tube types 26, 2A3, 45, and 6J5GT had electrodes which were approximately plane, while types 76 and 6AG7 had cylindrical electrodes. The use of the theoretical formula for cylindrical arrangements appeared legitimate since the characteristics obtained with tubes of different geometry could be reduced to a common pattern, as is best shown in Fig. 6.

(e) It is quite likely that contact potentials existed within the tubes. Some attempts at matching the theoretical and experimental curves by assuming a small contact potential were made. This procedure met without any great degree of success, for in order to provide any closer agreement by this means the contact potential would have to vary with the grid voltage. Since there was no logical basis for such an assumption, attempts at correction by this method were abandoned.

SUGGESTIONS FOR IMPROVEMENT

This work is by no means intended as a complete verification of the Jaffé theory. Rather, it is preliminary and indicates the direction which further investigation should take. Vacuum tubes used in the experimental work were those readily available, but their construction in no sense approximated the ideal conditions of the theory. It is suggested that succeeding experimental evidence be based upon tubes of special construction, with particular emphasis upon the structure and spacing of the tube elements. Effects due to secondary emission should be reduced to a minimum by careful selection of the tube electrodes and treatment of their surfaces.

On the other hand, an attempt should be made to modify the theory in such a way that it takes into account more realistically the conditions in actual discharge tubes. It is therefore desirable to extend the derivation not only to the cylindrical arrangement, but also to the case where the grid is treated as an actual grid, allowing for the nonhomogeneity of the field near the grid, and for penetration of the field.

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[3] The possibility of secondary emission taking place under such conditions as these is confirmed, for example, in the following reference: Sir J. J. Thomson and C. P. Thomson, "Conduction of Electricity through Gases," vol. 2, University Press, Cambridge, 1933, pp. 183, 187, 188, 189, 190.

[4] H. F. Dart, "Vacuum-Tube Testing and Design," International Textbook Co., Scranton, Pa., 1939; pp. 15, 45.